UNDERGRADUATE SIXTH SEMESTER (PROGRAMME) EXAMINATIONS, 2022

Subject: Mathematics **Course ID: 62118**

Course Code: SP/MTH/601/DSE-1B **Course Title: Probability and Statistics**

Time: 2 hours **Full Marks: 40**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

1. Answer *any five* of the following questions: **(2 X 5 = 10)**

a) If *A* and *B* are two independent events, then prove that $\bar{A}$ and $\bar{B}$ are also independent.

b) Define probability density function of a random variable *X*.

c) Find the value of the constant *K* such that

$$f(x) = \begin{cases} Kx(1-x), & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$$

is a possible probability density function.

d) Prove that $Var(X) = E\{X(X-1)\} - E(X)\{E(X) - 1\}$.

e) From a normal population with standard deviation 3, nine observations are drawn: 25, 20, 18, 27, 23, 32, 19, 21, 22. Obtain the 95% confidence interval for the population mean. [For a standard normal variate $z, P(-1.96 \le z \le 1.96) = 0.95$].

f) Find the moment generating function (m.g.f.) of Poisson distribution.

g) What do you mean by Markov Chain?

h) State Weak Law of Large Numbers (WLLN).

2. Answer *any four* of the following questions: **(5 X 4 = 20)**

a) **(i)** State the axiomatic definition of probability.

(ii) If the probability density function $f(x)$ of a random variable *X*is defined by

$$f(x) = \frac{1}{2}e^{-|x|}, \quad -\infty < x < \infty,$$

Calculate mean and variance. (2+3)

b) (i) Prove that $|E\{g(X)\}| \le E\{|g(X)|\}$, for continuous random variable *X.*

(ii) A coin is tossed repeatedly until a head is obtained. If the tosses are independent and probability of head is p for each toss, find the expected number of tosses. (2+3)

c) The joint density function of the random variable *X* and *Y* is given by

$$f(x, y) = 2, 0 < x < 1, 0 < y < x$$

Find the marginal and conditional density functions. Compute $P\left(\frac{1}{4} < X < \frac{3}{4} \mid Y = \frac{1}{2}\right)$.

d) (i) Define Mathematical expectation for Bi-variate distribution.

(ii) Find the correlation coefficient of (2X – 3) and (X + 2). [2+3]

e) Find the mean and variance of the Binomial distribution.

f) Show that the function $f(x, y)$ defined by

$$f(x,y) = \begin{cases} \sin x \sin y, & 0 < x < \frac{\pi}{2}, 0 < x < \frac{\pi}{2} \\ 0, & elsewhere \end{cases}$$

is a possible two dimensional probability density function. Find the marginal density function and prove that the random variables are independent.

3. Answer *any one* of the following questions: (10 X 1 = 10)

a) **(i)**Find the maximum likelihood estimator of the parameter of a Poisson distribution.

(ii) Use Chebyshev's inequality to show that$P\left\{\left|X - \frac{1}{2}\right| \leq \frac{2}{\sqrt{12}}\right\} \geq \frac{3}{4}$ for a random variable having probability density function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & elsewhere. \end{cases}$ (5+5)

b) **(i)**If *X* is a normal *N*(0, σ) variate, then find the variance of $(X + X^2)$.

(ii) The regression equation of two random variables *X* and *Y* are $2x + 3y = 5, 5x + 2y = 8$.Find the correlation coefficient.

(iii) Prove that $\mu_3 = \alpha_3 - 3m\alpha_2 + 2m^3$. (4+3+3)

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