

# B.Sc. 5th Semester (Programme) Examination, 2020-21

**Subject: PHYSICS**

**Course ID: 52418**

**Course Code : SP/PHS/501/DSE-1A**

Course Title: Advanced Mathematical Physics

**Time: 2 Hour**

**Full Marks: 40**

*The figures in the right hand side margin indicate full marks.  
Candidates are required to give their answers in their own words  
as far as practicable.*

**1. Answer any five of the following questions: (2×5=10)**

- Write down some applications of Gaussian Distribution.
- Show that the gradient of a scalar field is a covariant vector.
- Discuss kernel of a function.
- Show that any tensor of rank 2 can be expressed as the sum of a symmetric and anti-symmetric tensors of rank 2.
- Show that  $\frac{\partial x^p}{\partial x^q} = \delta_q^p$ .
- Prove that Kronecker delta is a mixed tensor of rank 2.
- Let  $G$  be an abelian group. Prove that the subset  $H = \{g \in G: g^2 = e \text{ (identity element)}\}$  forms a subgroup of  $G$ .
- There are 3 arrangements of the word DAD, namely DAD, ADD, and DDA. How many arrangements are there of the word PROBABILITY?

**2. Answer any four of the following questions: (5×4=20)**

- Discuss briefly homomorphism and isomorphism of groups.
- Define stress, strain and elasticity tensors.
- Show that the set  $Z$  of all integers form a group with respect to binary operation  $*$

defined by  $a * b = a + b + 1 \quad \forall a, b \in Z$

is an abelian group.

(d) (i) Explain continuous and discrete probability distributions. (ii) State Baye's theorem. 3+2=5

(e) Prove that the angles  $\theta_{12}$ ,  $\theta_{23}$  and  $\theta_{31}$  between the coordinates curves in a three dimensional coordinate system are given by

$$\cos \theta_{12} = \frac{g_{12}}{\sqrt{g_{11}g_{22}}}, \quad \cos \theta_{23} = \frac{g_{23}}{\sqrt{g_{22}g_{33}}}, \quad \cos \theta_{31} = \frac{g_{31}}{\sqrt{g_{33}g_{11}}}.$$

(f) Two dice are rolled

A= 'sum of two dice equals 3'

B= 'sum of two dice equals 7'

C= 'at least one of the dice shows a 1'

(a) What is  $P(A|C)$  ?

(b) What is  $P(B|C)$

(c) Are A and C independent? 2+2+1=5

**3. Answer any one of the following questions:**

**(10×1=10)**

a) (i) Distinguished between symmetric and antisymmetric tensors. (ii) What do you mean by contraction of a tensor? (iii) A covariant tensor has components  $xy, 2y-z^2, xz$  in rectangular co-ordinates. Find its covariant components in spherical co-ordinates.

(iv) What is Einstein's summation convention?

4+2+3+1=10

b) Let  $X \sim N(\mu, \sigma^2)$  for some  $\mu \in \mathbf{R}, \sigma \in \mathbf{R} > 0$ , where N is the Gaussian distribution. Prove that  $var(X) = \sigma^2$ .

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