SP-V/Math-501-DSE-1A/19

B.Sc. 5th Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 52118 **Course Code : SPMTH-501-DSE-1A**

Course Title : Theory of Equations

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* of the following: 2×5=10

(a) Find the quotient and remainder when $x^4+5x^3+4x^2+8x-2$ is divided by $x+2$.

(b) Apply Decarte's rule of signs to find the nature of the roots of the equation $x^4+2x^2+3x-1=0$.

(c) Prove that $x^{40}+x^{23}+x^{30}+x^{13}$ is divisible by x^2+1.

(d) If α,β,γ be the roots of the cubic equation $x^3+px^2+qx+r=0$, find the value of $\Sigma\,\alpha^2\beta$.

(e) If a, b are the roots of the equation $x^2-px+q=0$, then find the equation whose roots are $\frac{1}{a},\frac{1}{b}$.

(f) Form a cubic equation whose roots are 2, 3 – 2*i*.

(g) Verify that $(x+1)^4+x^4+1=0$ is a reciprocal equation.

(h) If α be an imaginary root of the equation $x^n-1=0$, where *n* is a prime number, prove that $(1-\alpha)(1-\alpha^2)\cdots(1-\alpha^{n-1})=n$.

2. Answer *any four* questions from the following: 5×4=20

(a) State the fundamental theorem of classical algebra. If the equation $x^4+px^2+qx+r=0$ has three equal roots then show that $8p^3+27q^2=0$ and $p^2+12r=0$.

(b) If α,β,γ be the roots of the equation $x^3+3x+1=0$, find an equation whose roots are $\frac{\alpha}{\beta}+\frac{\beta}{\alpha},\frac{\beta}{\gamma}+\frac{\gamma}{\beta},\frac{\gamma}{\alpha}+\frac{\alpha}{\gamma}$.

(c) If α be a root of the equation $x^3-3x-1=0$, prove that the other roots are $2-\alpha^2,\alpha^2-\alpha-2$.

(d) If α,β,γ are the roots of the equation $x^3+3x^2+10=0$, prove that $\alpha^4+\beta^4+\gamma^4=201$.

(e) If α is a special root of the equation $x^8-1=0$, then show that $(\alpha+2)(\alpha^2+2)\cdots(\alpha^7+2)=85$.

(f) Reduce the equation, $6x^6+25x^5+31x^4-31x^2-25x-6=0$ to a reciprocal equation of the standard form and then solve it.

3. Answer *any one* question from the following: 10×1=10

(a) (i) Use Sturm's theorem to show that $x^3 - 7x + 7 = 0$ has two roots between 1 and 2 and the other root between – 3 and – 4.

(ii) Solve by Cardan's method, the equation $x^3 - 6x^2 - 6x - 7 = 0$. 5+5=10

(b) (i) Prove that $f(x) = 0$ be a reciprocal equation of degree *n* and of the first type if and only if $f(x) = x^n f\left(\frac{1}{x}\right)$.

(ii) If α be an imaginary root of the equation $x^7 - 1 = 0$, find the equation whose roots are $\alpha + \alpha^6, \alpha^2 + \alpha^5, \alpha^3 + \alpha^4$. 5+5=10

B.Sc. 5th Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 52118 **Course Code : SPMTH-501-DSE-1A**

Course Title : Linear Programming

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* of the following: 2×5=10

(a) Define Basic feasible solution and optimal solution.

(b) Examine if $X = \{(x, y) : |X| \leq 2\}$ is a convex set.

(c) Prove that $x_1 = 2, x_2 = 3, x_3 = 0$ is a feasible solution but not basic solution of the set of equations $3x_1 + 5x_2 - 7x_3 = 21; 6x_1 + 10x_2 + 3x_3 = 42$.

(d) Apply maximum and minimum principle to solve the game whose pay-off matrices are

$$\begin{pmatrix} 15 & 2 & 3 \\ 6 & 5 & 7 \\ -7 & 4 & 0 \end{pmatrix}$$

(e) What is the condition for optimality and entering variable in simplex table?

(f) What is two-person zero sum game? Give a suitable example.

(g) Write down the mathematical formulation of an assignment problem.

(h) State fundamental theorem of duality.

2. Answer *any four* questions from the following: 5×4=20

(a) Food X contains 6 units of vitamin A and 7 units of vitamin B per gram and costs 12P./gm. Food Y contains 8 units and 12 units of A and B per gram respectively and costs 20 p./gm. The daily requirement of vitamin A and B are at least 100 units and 120 units respectively. Formulate the above as an L.P.P. to minimize the cost. Then solve it graphically.

(b) Use duality to solve the following L.P.P.

Minimize $Z = 3x_1 + x_2$

Subject to $2x_1 + 3x_2 \geq 2$

$x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0.$

(c) Solve the following game graphically.

		Player B		
		B_1	B_2	B_3
Player-A	A_1	1	3	11
	A_2	8	5	2

(d) Define convex set. Show that the set of all feasible solution of a L.P.P. is a convex set. 1+4=5

(e) Solve the following L.P.P. graphically:

Maximize $Z = -x + 2y$.

Subject to $-x + y \leq 1; -x + 2y \leq 4; x, y \geq 0$.

(f) Find the optimal solution on the transportation problem

2	2	2	1	3
10	8	5	4	7
7	6	6	8	5
4	3	4	4	

3. Answer *any one* question: 10×1=10

(a) (i) $x_1 = 1$ $x_2 = 2$, $x_3 = 1$ and $x_4 = 0$ is a feasible solution to the set

$$11x_1 + 2x_2 - 9x_3 + 4x_4 = 6$$

$$15x_1 + 3x_2 - 12x_3 + 5x_4 = 9$$

Reduce F.S. to one B.F.S.

(ii) Solve the following transportation problem by Matrix minima method.

2	2	3	10
4	1	2	15
1	3	1	40
20	15	30	

(iii) Define saddle point. 4+4+2=10

(b) (i) Solve the following L.P.P. by using two phase simplex method

Minimize $Z = x_1 - 2x_2 - 3x_3$

Subject to
$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1, \quad x_1, x_2, x_3 \geq 0$$

(ii) If we add a fixed number to each element of a pay-off matrix, the optimal strategies remain unchanged but the value of the game is increased by that number. 5+5=10

———