B.SC. FIFTH SEMESTER (PROGRAMME) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 52118**

Course Code: SP/MTH/501/DSE-1A **Course Title: Theory of Equations**

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

1. Answer *any five* questions: 2 x 5=10

   (a) If two roots of a polynomial equation with rational coefficients of degree four are $2 + 3i$, $2 - \sqrt{3}$, then find the equation.

   (b) Apply Descartes' rule of signs to find the nature of roots of the equation $x^4 + 7x^2 + 5x - 4 = 0$.

   (c) Find the quotient and remainder when $x^5 - 4x^4 + 3x^3 - 40x^2 - 20x + 41$ is divided by $x - 5$.

   (d) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find $\Sigma\alpha 2$.

   (e) If α is a double root of the equation $ax^3 + 3bx^2 + 3cx + d = 0$, find the value of α.

   (f) Find the condition such that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in geometric progression.

   (g) Determine the multiple roots of $x^5 + 2x^4 + 2x^3 + 4x^2 + x + 2 = 0$.

   (h) Find the special roots of $x^{12} - 1 = 0$.

2. Answer *any four* questions: 5x4=20

   (a) Solve the equation $2x^3 + x^2 - 5x + 2 = 0$, if two of its roots α and β are connected by the relation $\alpha\beta + 1 = 0$.

   (b) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\Sigma\alpha^3\beta^3$.

   (c) If α, β, γ are the roots of $ax^3 + bx^2cx + d = 0, d \neq 0$, find the equation whose roots are $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}, \gamma + \frac{1}{\gamma}$.

   (d) Solve the reciprocal equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

   (e) Show that the roots of the equation $\frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3} = x$ are all real.

   (f) (i) State the fundamental theorem of classical algebra.

   (ii) Find the condition for the equation $X^3 + 3HX + G = 0$ to have three distinct real roots.

3. Answer *any one* question: 10x1=10

   (a) (i) Solve the equation $3x^3 - 22x^2 + 48x - 32 = 0$, the roots of which are in harmonic progression. 5

(ii) Find the value of k for which the equation $x^4 + 4x^3 - 2x^2 - 12x + k = 0$ has four real and unequal roots. 5

(b) (i) If α is a root of the cubic $x^3 - 3x + 1 = 0$, then show that the other roots are $\alpha^2 - 2$ and $2 - \alpha - \alpha^2$. 5

(ii) Solve by Ferrari's method: $x^4 - 9x^3 + 28x^2 - 38x + 24 = 0$. 5

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