B.SC. FIFTH SEMESTER (PROGRAMME) EXAMINATION, 2021

Subject: Mathematics **Course ID: 52118**

Course Code: SP/MTH/501/DSE-1A **Course Title: Linear Programming**

Time 2 Hours **Full Marks: 40**

<u>The figures in the margin indicate full marks</u>

Notations and symbols have their usual meaning

1. Answer any five questions: $2 \times 5 = 10$

(a) Define feasible solution and optimal solution of an LPP.

(b) Convert the following constraints into equations by using slack or surplus variables

$$|2x_1 - 3x_2 + x_3| \le 24$$

(c) Prove that $x_1 = 2, x_2 = -1, x_3 = 0$ is a basic solution to set of equations

$$4x_1 - x_2 + 2x_3 = 9$$
$$2x_1 - 3x_2 + 7x_3 = 7$$

(d) Determine the position of the points (1,0,2,-2) and (0,4,1,0) with respect for the hyperplane

$$2x_1 + 3x_2 + x_3 - 3x_4 = 13$$

(e) Find the dual of the LPP

$$\text{Maximize } z = 4x_1 + 3x_2$$
$$\text{Subject to } x_1 + x_2 \le 5$$
$$2x_1 - 3x_2 \le 2, \quad x_1, x_2 \ge 0$$

(f) Write down the mathematical formulation of an assignment problem.

(g) In a transportation problem with 3 origins and 4 destinations, can the variables $x_{11}, x_{12}, x_{23}, x_{24}, x_{31}, x_{34}$ be considered as basic variables? Give reason.

(h) Solve the following game problem and find the value of the game

-4	6
2	-3

2. Answer any four questions: $5 \times 4 = 20$

(a) Following is the starting tableau of an LPP by the simplex method (for a maximization problem), in an incomplete form

			c_j							
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7
0	a_5	x_5	20	-4	6	5	-4			
0	a_6	x_6	10	3	-2	4	1			
0	a_7	x_7	20	8	-3	3	2			
$(z_j - c_j)$				-4	-1	-3	-5	0	0	0

(i) Complete the objective row and the tableau.

(ii) Write down the LPP in its standard form from the tableau.

(iii) Write down the actual problem.

(iv) Find the departing and the entering vectors and write down the next tableau. 1+1+1+2

(b) (i) Solve the LPP by graphical method 3

$$\begin{aligned} \text{Min } z &= 4x_1 + 2x_2 \\ \text{S.to } 3x_1 + x_2 &\geq 27 \\ -x_1 - x_2 &\leq -21 \\ x_1 + 2x_2 &\geq 30 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(ii) Prove that the set $S = \{(x, y) : x^2 + y^2 = 4\}$ is not a convex set. 2

(c) Solve by "Big M-method" Max $z = x_1 - 2x_2 + 3x_3$

$$\begin{aligned} \text{Sub. to } \; x_1 + 2x_2 + 3x_3 &= 15 \\ 2x_1 + x_2 + 5x_3 &= 20 \end{aligned}$$

where $x_1, x_2 \geq 0$.

(d) Find the optimal solution of the following problem by solving its dual.

$$\begin{aligned} \text{Max } z &= 3x_1 + 4x_2 \\ \text{S.to } x_1 + x_2 &\leq 10 \\ 2x_1 + 3x_2 &\leq 18 \\ x_1 &\leq 8 \\ x_2 &\leq 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(e) Find the optimal assignment to find the minimum cost for the assignment problems with the following cost matrix

	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

(f) (i) Solve the following game by linear programming technique 4

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{array}{|ccc|} \hline 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \\ \hline \end{array} \end{array}$$

(ii) Define saddle point of a game. 1

3. Answer any one: $10 \times 1 = 10$

(a) (i) Show that X= $\{(X_1,X_2): 9X_1^2 + 4X_2^2 \leq 36\}$ is a Convex set.

(ii) Solve by simplex method

$$\begin{aligned} &\text{Maximize} \quad z = 4x_1 + 14x_2 \\ &\text{Subject to } 2x_1 + 7x_2 \leq 21 \\ &\qquad\qquad 7x_1 + 2x_2 \leq 21, \\ &\qquad\qquad x_1, x_2 \geq 0 \end{aligned}$$

Will the above LPP admit an alternative solution? Give reason.

If yes then find another solution of the above LPP. 2+(5+3)=10

(b) (i) Transform the games with the following pay off matrix to corresponding LPP

$$\begin{array}{|ccc|} \hline 2 & -2 & 3 \\ -3 & 5 & -1 \\ \hline \end{array}$$

What is a zero sum game?

(ii) Show that the 2×2 game $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ is non-strictly determined, if $p < q, p < r, s < q$ and $s < r$.

(iii) Prove that in LPP, the dual of the dual is Primal. 4+3+3=10

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