B.SC. FIFTH SEMESTER (HONS.) EXAMINATION 2021

Subject: Mathematics **Course ID: 52117**

Course Title: Boolean Algebra and Automata **Course Code: SH/MTH/504/DSE- 2**

Full Marks: 40 **Time: 2 hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning.

1. Answer *any five* of the following questions: $(\mathbf{2 \times 5 = 10})$

a) Define down-sets and up-sets.

b) State the Knaster–Tarski Fixed point Theorem for a complete lattice.

c) Design context-free grammars for the following language: The set of all strings with twice as many 0's as 1's.

d) Write the formal definition of Pushdown Automata.

e) Show that if P is a Pushdown Automata, then there is a one-state Pushdown Automata P' such that $N(P') = N(P)$.

f) Define the language of a Turing Machine.

g) How do you define a Recursive Language?

h) Give an example of a Turing Machine that accepts the Empty Language.

2. Answer *any four* of the following questions: $(\mathbf{5 \times 4 = 20})$

a) Let P and Q be chains. Prove that $P \times Q$ is a chain in the lexicographic order. Prove that $P \times Q$ is a chain in the coordinate-wise order if and only if at most one of P and Q has more than oneelement.

b) Prove that, for all ordered sets *P*, *Q* and *R*, $< P \longrightarrow < Q \longrightarrow R >> \cong < P \times Q \longrightarrow R >$.

c) Let L be a lattice. Prove that the following are equivalent:

(i) L is a chain;

(ii) every non-empty subset of L is a sublattice;

(iii) every two-element subset of L is a sublattice.

d) A subset A of N is called co-finite if $N - A$is finite.

(i) Show that the collection of all co-finite subsets of Nforms a lattice.

(ii) Show that the collection of all subsets of N which are eitherfinite or co-finite forms a lattice.

e) Show that every regular language is a context-free language.

f) Show that, given a TM that computes f, you can construct a TM that accepts the graph of f as a language.

g) Show that the set of Turing-machine codes for TM's that accept all inputs that are palindromes (possibly along with some other inputs) is undecidable.

3. Answer *any one* of the following questions: $\mathbf{(10 \times 1 = 10)}$

a) Let $A = (a_{ij})$ be an $m \times n$ matrix whose entries are elements of alattice L.

(i) Prove the Mini-Max Theorem, viz.$\bigvee_{j=1}^{n}(\bigwedge_{i=1}^{m} a_{ij}) \leq \bigwedge_{k=1}^{m}(\bigvee_{l=1}^{n} a_{kl})$, that is, (the join of the meets of the columns of A) ≤ (themeet of the joins of the rows of A).

(ii) By applying (i) to a suitable 2 × 2 matrix, derive the distributive inequality$a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$.

(iii) By applying (i) to a suitable 3×3 matrix, derive the median in equality$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$. 4+3+3

b) (i) Draw switching circuits for these Boolean expressions:

(A) $(a \vee b) \wedge (b \vee c) \wedge (c \vee a)$.

(B) $[a \wedge ((b \wedge \sim c) \vee (\sim b \wedge c))] \vee (\sim a \wedge b \wedge c)$

(ii) Establish a truth table for the Boolean function $f(x_1, x_2, x_3) = (\sim x_1 \vee x_2) \wedge (\sim x_3 \vee x_2)$.

Draw a circuit using as few AND, OR, NOT gates as possible to model the function. 4+6

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