B.SC. FIFTH SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 52117**

Course Code: SH/MTH/504/DSE-2 **Course Title: Probability and Statistics**

Full Marks: 40 **Time: 2 Hours**

<u>The figures in the margin indicate full marks</u>

Notations and symbols have their usual meaning

1. **Answer *any five* of the following questions:** **$(2 \times 5 = 10)$**

   (a) Let the cumulative distribution function of a random variable is given by

   $$F(x) = \begin{cases} 1 - \frac{1}{4}e^{-x}, & x \geq 0 \\ 0, & \text{otherwise}. \end{cases}$$

   (b) Show that the jump of the distribution function at a point is the probability at that point.

   (c) Give an example to show that two events can be independent but not mutually exclusive.

   (d) Prove that probability distribution function is a monotonically non-decreasing function.

   (e) Find the value of the constant k such that the function $f(x,y)$ is a possible joint probability density function of the random variables X & Y where

   $$f(x,y) = \begin{cases} \frac{k(2-x-y)}{3}, & \text{for } x > 0, y > 0, x + y < 2 \\ 0, & \text{elsewhere} \end{cases}.$$

   (f) Prove that $Var(X) = E(X^2) - \{E(X)\}^2$.

   (g) Show that the second order moment about any point is minimum when taken about the mean.

   (h) If X is a $\gamma(n)$ variate, then show that $P(0 < X < 2n) \geq \frac{n-1}{n}$.

2. **Answer *any four of* the following questions:** **$5 \times 4 = 20$**

   (a) (i) Let X be a continuous random variable with distribution function $F(x)$ and probability density function $f(x)$. Show that $F(x) = \int_{-\infty}^{x} f(x)dx$.

   (ii) Let X be a random variable with pdf $f(x) = \begin{cases} 7e^{-7x}, & x > 0 \\ 0, & otherwise \end{cases}$.

   Find pdf of $Y = X^{1/3}$. 2+3

   (b) (i) Find the median of X with pdf f, given by $f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$.

(ii) Show that for continuous symmetrical distribution with unique median, the median and mean are equal. 2+3

(c) For the probability density function

$$f(x,y) = 3x^2 - 8xy + 6y^2, 0 < x < 1, 0 < y < 1$$

Find the regression curves for the meanand also the least square regression lines. 2+3

(d) A random variable X has probability density function $12\, x^2(1-x)(0 < x < 1)$. Compute $P(|X - m| \geq 2\sigma)$, and compare it with the limit given by Tchebycheff's Inequality. 5

(e) Obtain the maximum likelihood estimator of the parameter of a Poisson distribution.

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(f) For the two state Markov chain with transition matrix $\begin{pmatrix} p & 1-p \\ q & 1-q \end{pmatrix}$ and initial probability distribution(π_1, π_2), calculate the probability distribution at the n^{th}trial and show that as$n \to \infty$, this distribution is independent of the initial distribution. 2+3

3. Answer *any one* of the following questions: **$(10 \times 1 = 10)$**

a) i) Find confidence interval for m of normal (m, σ) population when $\sigma$is known.

ii) Obtain Bernoulli's theorem as a particular case of the law of large numbers. 5+5

b) (i) The mean and variance of a sample of size 400 from a normal population are found to be 18.35 and 3.25 respectively. Given $P(U > 1.96) = 0.025$, U being a standard normal variate. Find 95% confidence interval for the population mean.

(ii) If X and Y be two standardized random variables and $\rho(aX + bY, bX + aY) = \frac{1+2a}{a^2+b^2}$, then find the correlation coefficient $\rho(X, Y)$ between X and Y. 5+5

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