B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52116 **Course Code : SHMTH-503-DSE-1**

Course Title: Linear Programming

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mention, symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Justify the statement that vertex is a boundary point, but all boundary points are not vertices.

(b) Write the dual of Minimize $6x_1 + 3x_2$, subject to $3x_1 + 4x_2 + x_3 \geq 5$; $6x_1 - 3x_2 + x_3 \geq 2$; $x_i \geq 0$, $i = 1, 2, 3$.

(c) When an LPP has multiple solution, answer in the context of simplex method.

(d) Determine the convex hull of the set $A = \{(x_1, x_2)\ ;\ x_1^2 + x_2^2 = 1\}$.

(e) Is $(2, 0, 0, 1, 0)$ a basic feasible solution of the system of equations

$2x_1 + 2x_2 - x_3 + x_4 + 5x_5 = 5$, $4x_1 + 3x_2 - 3x_3 + 2x_4 - 3x_5 = 10$? Justify.

(f) Write down the mathematical formulation of an assignment problem.

(g) For what value of λ the game with following pay-off matrix is strictly determinable?

	B_1	B_2	B_3
A_1	λ	7	5
A_2	–2	λ	–8
A_3	–3	4	λ

(h) In a transportation problem with 3 origins and 4 destinations, can the variables x_{11}, x_{12}, x_{22}, x_{34}, x_{23}, x_{13} be considered as basic variables? Give reason.

2. Answer *any four* questions: 5×4=20

(a) The first tableau of an L.P.P. by simplex method is given below (for a maximization problem) in an incomplete form:

C_j										
C_B	B	X_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7
0	a_5	x_5	20	– 4	6	5	– 4			
0	a_6	x_6	10	3	– 2	4	1			
0	a_7	x_7	20	8	– 3	3	2			
$Z_j - C_j$				– 4	– 1	– 3	– 5	0	0	0

(i) Complete the objective row and the tableau. 1

(ii) Write down the L.P.P. in its standard form. 1

(iii) Write down the actual problem. 1

(iv) Find the departing and the entering vector and write down the next tableau. 1+1=2

(b) By solving the dual of the following problem, show that the given problem has no feasible solution. 5

Minimize $Z = x_1 - x_2$

subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0.$

(c) Use Big-M method to solve the following L.P.P.: 5

Maximize $Z = 3x_1 - x_2$

subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0$.

(d) Solve the following assignment problem with the cost matrix: 5

	M_1	M_2	M_2	M_4	M_5
I	5	11	10	12	4
II	2	4	6	3	5
III	3	12	5	14	6
IV	6	14	4	11	7
V	7	9	8	12	5

and find the minimum cost of assisgnment,

(e) Find the optimal solution and the corresponding cost of transportation in the following transportation problem: 5

	D_1	D_2	D_3	D_4	a_i
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_i	5	8	7	14	

(f) Solve graphically the game whose pay-off matrix is

$$\begin{pmatrix} 2 & 2 & 3 & -1 \\ 4 & 3 & 2 & 6 \end{pmatrix}.$$

3. Answer *any one* questions: 10×1=10

(a) (i) Prove that there are only $(m+n-1)$ independent equations in a T.P., m, n being the number of origin and destinations and only one equation can be dropped as being redundant. 3

(ii) Show that set of all feasible solutions of an L.P.P. is convex set. 2

(iii) Using simplex method, obtain the inverse of the matrix $= \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$. 5

(b) (i) Prove that dual of the dual is the primal. 3

(ii) In a rectangular game the pay-off matrix A is given by

$$A = \begin{pmatrix} 3 & 2 & -1 \\ 4 & 0 & 5 \\ -1 & 3 & -2 \end{pmatrix}.$$

State giving reasons whether the payers will use pure or mixed strategies. What is the value of the game? 5

(iii) Solve the following assignment problem with the cost matrix

5	11	10	12	4
2	4	6	3	5
3	12	5	14	6
6	14	4	11	7
7	9	8	12	5

and find the minimum cost of Assignment. 2

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B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52116 **Course Code : SHMTH-503-DSE-1**

Course Title: Theory of Equations

Time: 2 Hours **Full Marks: 40**

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Candidates are required to give their answers in their own words as far as practicable.

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1. Answer *any five* questions: 2×5=10

(a) Find the value of a such that $x^4+3x^3+x^2+ax+3$ is exactly divisible by $x+1$.

(b) From a biquadratic equation with rational coefficients, two of whose roots are $2i \pm 1$.

(c) $x^3+3px+q$ has a factor of the form $(x-\alpha)^2$. Show that $q^2+4p^3=0$.

(d) State Descartes's rule of signs. Using this rule find the minimum number of complex roots of the equation $x^5-4x^3+3x-9=0$.

(e) Find an upper limit of the real roots of the equation $x^4-2x^3+3x^2-2x+2=0$.

(f) If α be a multiple root of order 3 of the equation $x^4+bx^2+cx+d=0$ $(d \neq 0)$, show that

$$\alpha = -\frac{8d}{3c}.$$

(g) Find the condition that the equation $x^3+px^2+qx+r=0$ may have two roots equal but of opposite signs.

(h) Find the maximum and minimum values of the polynomial, if exist

$$f(x)=3x^4-16x^3+6x^2-48x+7.$$

2. Answer *any four* questions: 5×4=20

(a) (i) If the equation $x^4+ax^3+bx^2+cx+d=0$ have three equal roots, prove that each of them is equal to

$$\frac{6c-ab}{3a^2-8b}.$$

(ii) If the roots of the equation $x^3+px^2+qx+r=0$ are in A.P., show that $p^2 \geq 3q$. 3+2=5

(b) (i) Show that the condition of sum of two roots of the equation $x^4+mx^2+nx+p=0$ be equal to the product of the other two roots is

$$(2p-n)^2=(p-n)(p+m-n)^2.$$

(ii) If α, β, γ be the roots of the equation $x^3 + qx + r = 0$, find the value of $\sum \frac{1}{\alpha^2 - \beta\gamma}$. 3+2=5

(c) Find the relation among the coefficients of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, if its roots $\alpha, \beta, \gamma, \delta$ be connected by the relation $\alpha + \beta = \gamma + \delta$. 5

(d) If α, β, γ be the roots of the equation $x^3 + 3x + 1 = 0$, find the equation whose roots are

$\frac{1}{\beta^2} + \frac{1}{\gamma^2} - \frac{1}{\alpha^2}, \frac{1}{\gamma^2} + \frac{1}{\alpha^2} - \frac{1}{\beta^2}, \frac{1}{\alpha^2} + \frac{1}{\beta^2} - \frac{1}{\gamma^2}$. 5

(e) Show that if the roots of the equation $x^4 + x^3 - 4x^2 - 3x + 3 = 0$ are increased by 2, the transformed equation is a reciprocal equation. Solve the reciprocal equation and hence obtain the solution of the given equation. 5

(f) Express $f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$ in the form $(x^2 - 5x + \lambda)^2 - (ax + b)^2$. Hence solve $f(x) = 0$. 5

3. Answer *any one* question: 10×1=10

(a) (i) Apply Sturm's theorem to find the number and position of the real roots of the equation

$x^4 - 6x^3 + 10x^2 - 6x + 1 = 0$.

(ii) Show that roots of the equation

$\frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3} = \frac{1}{x}$ are all real. 6+4=10

(b) (i) Prove that the equation $(1 + x)^5 = a(1 + x^5)$ is a reciprocal one, where $a \neq 1$. Solve it when $a = 5$.

(ii) Prove that the special roots of the equation $x^9 - 1 = 0$ are the roots of the equation $x^6 + x^3 + 1 = 0$ and show that the roots of the equation $x^6 + x^3 + 1 = 0$ are $\alpha, \alpha^2, \alpha^4, \alpha^5, \alpha^7, \alpha^8$ where $\alpha = e^{\frac{2\pi i}{9}}$. (2+3)+(2+3)=10

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B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52116 **Course Code : SHMTH-503-DSE-1**

Course Title: Point Set Topology

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

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1. Answer *any five* questions: 2×5=10

(a) Find all compact subsets of a discrete metric space.

(b) Show that $\left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$ is compact in $\mathbb{R}$.

(c) Let A, B be compact subsets of a metric space X. Is $A \cap B$ compact?

(d) State Axiom of choice.

(e) Define a connected topological space.

(f) State Baire Category theorem.

(g) Let X have the discrete topology and Y is any topological space then show that any function $f : X \to Y$ is continuous.

(h) When is a finite subset of a metric space connected?

2. Answer *any four*: 5×4=20

(a) Prove that countable union of countable sets is countable. 5

(b) Let (X, d) be a metric space. Let us consider another metric $\bar{d}$ on X defined by $\bar{d}(x, y) = min\{1, d(x, y)\}$ for $x, y \in X$. Then prove that the metric topology on X induced by d is same as the metric topology on X induced by $\bar{d}$. 5

(c) (i) Let X be a discrete topological space. Determine the closure of any subset A of X. Also determine the dense subsets of X. 1+1

(ii) Let $f : X \to Y$ be any function. If (Y, τ_0) is an indiscrete space then show that

$f : (X, \tau) \to (Y, \tau_0)$ is continuous for any τ. 3

(d) (i) Show that the circle

$\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ is connected. 3

(ii) Is the space $\mathbb{R}^2 \backslash \{(0,0)\}$ connected? Justify. 2

(e) Show that continuous image of a compact metric space is compact.

(f) (i) What is the base of $\mathbb{R}$ endowed with the usual topology? 1

(ii) Write down a base for the discrete topology on X. 1

(iii) Show that all open intervals in $\mathbb{R}$ are homeomorphic. 3

3. Answer any one: 10×1=10

(a) (i) Let $X = \{a, b, c, d, e\}$ and let $A = \{\{a, b, c\}, \{c, d\}, \{d, e\}\}$. Find the topology on X generated by A. 3

(ii) Show that the usual topology U on the real line $\mathbb{R}$ is coarser than the upper limit topology T on $\mathbb{R}$ which has as a base the class of open-closed intervals $(a, b]$. 2

(iii) Let $f : X \to \mathbb{R}$ be a real continuous function defined on a connected set X. Then prove that f assumes as a value each member between any two of its values. 5

(b) (i) Let (X, τ) be a topological space and $\{A_\alpha\}$ be a family of connected subspaces of X. Prove that if $\bigcap_{\alpha \in A} A_\alpha \neq \phi$ then $\bigcup_{\alpha \in A} A_\alpha$is connected. 5

(ii) Prove that every complete metric space is of second category. 5

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