B.SC. FIFTH SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 52116**

Course Code: SH/MTH/503/DSE-1 **Course Title: Point Set Topology**

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

1. Answer *any five* of the following questions: (2 x 5 = 10)

a) Prove that $\mathcal{T} = \{X, \emptyset, \{a, c\}, \{a, b, c\}\}$ is a topology for $X = \{a, b, c, d\}$ and find all $\mathcal{T}$-closed subsets of *X*.

b) Let $X = \{a, b, c\}$, $\mathcal{T} = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$. Find the limit points of the set $A = \{b, c\}$.

c) Let $X = \{a, b, c, d\}$ and $\mathcal{T} = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$. Let $f: X \to X$ be defined by $f(a) = b, f(b) = d, f(c) = b, f(d) = c$. Discuss the continuity of *f* at '*c*' and '*d*'.

d) If $a \in \mathbb{R}$, show that singleton set {*a*} is closed in the usual topology on $\mathbb{R}$.

e) If *A* and *B* are connected subsets of a space *X* such that $A \cap B \neq \emptyset$. Prove that $A \cup B$ is connected.

f) Show that *D* is a dense subset of *X* iff $Int\ (X - D) = \emptyset$.

g) Give an example to show that a compact subset of a topological space need not be closed.

h) If $\tau_1 = \{X, \emptyset, \{a\}, \{b, c\}\}$ is a topology on $X = \{a, b, c\}$ and $\tau_2 = \{Y, \emptyset, \{r\}, \{p, q\}\}$ is a topology on *Y*, test the continuity of the mapping $f: X \to Y$ given by $f(a) = p, f(b) = r, f(c) = q$.

2. Answer *any four* of the following questions: (5 x 4 = 20)

a) Prove that the continuous image of a compact space is compact.

b) (i) Show that if $(X, \mathcal{T}_1)$ is disconnected and $\mathcal{T}_2$ is finer than $\mathcal{T}_1$, then $(X, \mathcal{T}_2)$ is disconnected.

(ii) Prove by a counter example that connectedness is not a hereditary property. 3+2

c) (i) Prove that every separable metric space is second countable.

(ii) In a topological space (*X*, *τ*), prove that a subset *A* of *X* is open if and only if $Int(A) = A$. 3+2

d) (i) Show that every closed subset of a compact space is compact.

(ii) Give an example to show that $Int(A U B) \neq Int(A)\ U\ Int(B)$. 3+2

e) Show that a homeomorphic image of a second countable space is second countable.

f) If (X, τ_1) and (Y, τ_2) are two topological spaces then show that the product space $(X \times Y, \tau)$ is connected if and only if *X* and *Y* are connected.

g) (ii) Show that in any topological space, every derived set is closed. (3+2)

3. Answer *any one* of the following questions: (10 x 1 = 10)

a) (i) State and prove Baire Category theorem.

(ii) Let (X, d) be a metric space and $A \subset X$. If *A* is connected, open and closed, then show that *A* is a component of *X*.

(iii) If α and β are ordinal numbers and $\beta > 0$ then show that $\alpha + \beta > \alpha$. (4+3+3)

b) (i) Let (X, τ) and (Y, τ') be two topological spaces and *f* be a bijective mapping from *X* to *Y*. Then show that *f* is continuous and closed if and only if *f* is homeomorphism.

(ii) Prove that a subset of $\mathbb{R}$ is connected if it is an interval.

(iii) Let *X*,*Y* be two topological spaces and let $f : X \to Y$ be continuous. If *A* is a connected subset in *X* then prove that *f(A)* is connected in Y. (4+3+3)

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