**B.SC. FIFTH SEMESTER (HONOURS) EXAMINATIONS, 2021**

**Subject: Mathematics** **Course ID: 52116**

**Course Code: SH/MTH/503/DSE-1** **Course Title: Theory of Equations**

**Full Marks: 40** **Time: 2 Hours**

**The figures in the margin indicate full marks**

**Notations and symbols have their usual meaning**

1. Answer *any five* questions: 2 x 5=10

(a) Express the polynomial $x^3 + 2x^2 + 3$ in terms of $x + 1$.

(b) For what values of $n, x^n + 1$ is divisible by $x + 1$.

(c) Show that the special roots of the equation $x^{16} - 1 = 0$ are $cos\frac{r\pi}{8} \pm isin\frac{r\pi}{8}, r =$ 1,3,5,7.

(d) Show that the equation $x^4 - 14x^2 + 24x + k = 0$ has four real and unequal roots if $-11 < k < -8$.

(e) Find by synthetic division, the quotient and the remainder if $5x^5 - 3x^3 + x - 1$ is divided by $3x - 2$.

(f) Show that the equation $x^3 + 2x^2 - 2x - 1 = 0$ has a root in the open interval $(-3, -1)$.

(g) Find the remainder when the polynomial $8x^3 + 4x + 2$ is divided by $2x - 1$.

(h) Show that the equation $x^5 + 5ax^3 + 5a^2x + b = 0$ will have a pair of equal roots if $b^2 + 4a^5 = 0$.

(i) If α, β, γ, δ are the roots of the biquadratic equation $x^4 - x^3 + 2x^2 + x + 1 = 0$, then prove that $(\alpha^3 + 1)(\beta^3 + 1)(\gamma^3 + 1)(\delta^3 + 1) = 16$.

2. Answer *any four* questions: 5x4=20

(a) If $\alpha, \beta, \gamma$ are the roots of the equation $x^3 + qx + r = 0$, form the equation whose roots are $\frac{\alpha}{\beta}, \frac{\alpha}{\gamma}, \frac{\beta}{\alpha}, \frac{\beta}{\gamma}, \frac{\gamma}{\alpha}, \frac{\gamma}{\beta}$.

(b) If $\alpha, \beta, \gamma$ are the roots of the equation $x^3 - 9x + 9 = 0$, then show that $(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha) = \pm 27$.

(c) If $\alpha, \beta, \gamma$ are the roots of the equation $4x^3 - 8x^2 - 19x + 26 = 0$, find the equation whose roots are $\alpha - 2, \beta - 2, \gamma - 2$. Apply Descartes' rule of signs to both the equations to find the exact number of positive and negative roots. 2+3

(d) If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$, prove that $\sum(\alpha - \beta)^2 = 3p^2 - 8q$.

(e) If $\alpha_1, \alpha_2, \dots\dots\dots, \alpha_n$ are the roots of the equation $f(x) = 0$ where $f(x) = x^n + p_1x^{n-1} + p_2x^{n-2} + \cdots + p_n$, then prove that $f'(x) = \frac{f(x)}{x-\alpha_1} + \frac{f(x)}{x-\alpha_2} + \cdots + \frac{f(x)}{x-\alpha_n}$.

(f) Prove that the equation $x^5 + 1 + (x^3 + 1)(x^2 - x + 1) = 0$ is a reciprocal equation and solve it.

3. Answer *any one* question: 10x1=10

(a) (i) If $\alpha + \beta + \gamma = 1, \alpha^2 + \beta^2 + \gamma^2 = 3$ and $\alpha^3 + \beta^3 + \gamma^3 = 7$, prove that $\alpha^4 + \beta^4 + \gamma^4 = 11$. 3

(ii) If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, find the value of $\Sigma\alpha^3\beta\gamma$. 4

(iii) If $x^2 + kx + 1$ is a factor of $px^4 + qx^3 + r$, prove that $(p + r)(p - r)^2 = q^2 r$. 3

(b) (i) If $\alpha_1, \alpha_2, \dots\dots\dots, \alpha_n$ are the roots of the equation $x^n + nax + b = 0$, prove that $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3) \dots\dots\dots\dots (\alpha_1 - \alpha_n) = n({\alpha_1}^{n-1} + a)$. 3

(ii) Solve the bi-quadratic equation $x^4 + 3x + 20 = 0$, by Ferrari's method. 5

(iii) Transform the equation $x^4 + \frac{1}{2}x^3 - \frac{1}{3}x^2 + \frac{1}{4}x + \frac{5}{12} = 0$, so that the fractional co-efficients of the equation may be removed by multiplication of roots. 2

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