B.SC. FIFTH SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 52116**

Course Code: SH/MTH/503/DSE-1 **Course Title: Linear Programming**

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

1. **Answer any *five* questions** $2 \times 5 = 10$

a) What is the criteria for the existence of unique optimal solution in an LPP in a simplex method?

b) Prove that intersection of two convex sets is also a convex set.

c) Write the dual of $Maximize\ z = 6x_1 + 4x_2 + 6x_3 + x_4, subject\ to\ 4x_1 + 5x_2 + 4x_3 + 8x_4 = 21, 3x_1 + 7x_2 + 8x_3 + 2x_4 \leq 48, x_i \geq 0, i = 1,2,3,4$

d) When does an LPP admit an unbounded solution? Answer in the context of simplex method.

e) $x_1 = 4, x_2 = 0, x_3 = -2, x_4 = 0, x_5 = 2$ is a solution set of two linearly independent simultaneous equations with 5 variables. Is the solution basic? Give reason.

f) Write down the mathematical model of a transportation problem.

g) Use dominance to reduce the payoff matrices and solve the game

2	3	$\frac{1}{2}$
$\frac{3}{2}$	2	0
$\frac{1}{2}$	1	1

h) Show that whatever may be the value of a, the game with the following payoff matrix is strictly determinable.

		B	
		I	II
	I	3	7
A	II	-3	a

2. **Answer any four questions**: $5 \times 4 = 20$

(a) Following is the starting tableau of an LPP by the simplex method, in an incomplete form

			c_j	3	2	0	0	-M
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5
			2	2	1	1	0	0
			12	3	4	0	-1	1

(i) Complete the columns of C_B, B, x_B and the row giving $(z_j - c_j)$

(ii) Write down the LPP in its standard form from the tableau.

(iii) Find the departing and the entering vectors and write down the next tableau.

2+1+2

b) Apply simplex method to solve the following LPP

$$Maximize\ z = 30x_1 + 23x_2 + 29x_3,$$
$$subject\ to\ 6x_1 + 5x_2 + 3x_3 = 26,$$
$$4x_1 + 2x_2 + 5x_3 \leq 7,$$
$$x_i \geq 0, i = 1,2,3$$

From the final table find the optimal solution of dual problem.

c) Use Charne's Big-M method to solve the L.P.P

$$Max\ z = 3x_1 - x_2$$
$$S.to\ 2x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 3$$
$$x_2 \leq 4$$
$$x_1, x_2 \geq 0.$$

d) Solve the following assignment problem.

A company is faced with the problem of assigning six different machines to five different jobs. The costs are estimated in the adjacent table (hundred of rupees). Solve the problem assuming that the objective is to minimize the total cost

	1	2	3	4	5
1	2.5	5	1	5	1
2	2	5	1.5	7	3
3	3	6.5	2	8	3
4	3.5	7	2	9	4.5
5	4	7	3	9	6
6	6	9	5	10	6

e) Find the optimum B.F.S. of thetransportation problem

DESTINATION a_i

ORIGINS						a_i
	2	11	10	3	7	4
	1	4	7	2	1	8
	3	9	4	8	12	9
b_j	3	3	4	5	6	

f) Solve graphically or otherwise the games whose payoff matrices are given below

		B	
		B_1	B_2
A	A_1	1	-3
	A_2	3	5
	A_3	-1	6
	A_4	4	1
	A_5	2	2
	A_6	-5	0

3. **Answer any one question**: $10 \times 1 = 10$

a) (i) Solve the LPP by graphical method

$$Max\, z = 150\, x_1 + 100 x_2$$
$$S.to\; 8\, x_1 + 5\, x_2 \le 60$$
$$4\, x_1 + 5\, x_2 \le 40$$
$$x_1, x_2 \ge 0.$$

(ii) Prove that in LPP, the dual of the dual is Primal.

(iii) Solve the following LPP by using two-phase simplex method

$$Max\ \ z = 2\,x_1 - x_2 + x_3$$

$$S.to\ \ x_1 + x_2 - 3\,x_3 \le 8$$

$$4\,x_1 - x_2 + x_3 \ge 2$$

$$2\,x_1 + 3\,x_2 - x_3 \ge 4$$

$$x_1, x_2, x_3 \ge 0$$

3+2+5

b) i) If the dual problem has no feasible solution and the primal problem has a feasible solution,then prove that the primal objective function is unbounded.

ii) Find the optimal assignments to find the minimum cost for the assignment problems with the following cost matrix

	I	*II*	*III*	*IV*	*V*
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

iii) Show that the LPP

$$Maximize\ z = 4x_1 + 14x_2,$$

$$subject\ to\ 2x_1 + 7x_2 \le 21,$$

$$7x_1 + 2x_2 \le 21,$$

$$x_i \ge 0, i = 1,2$$

admits of an infinite number of solutions. 3+2+5

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