B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52111 **Course Code : SHMTH-501-C-11**

Course Title: Partial Differential Equation and Applications

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer *any five* questions : 2×5=10

(a) When a first order PDE is said to be Linear? Give an example of it.

(b) Form a partial differential equation by diminating constants A and p from $Z = Ae^{pt}\sin px$.

(c) Show that all the surfaces of revolution of the form $Z = f(x^2+y^2)$ with the Z-axis as the axis of symmetry, where f is an arbitrary function, satisfy the PDE.

$$yp - xq = 0, \quad \left(p = \frac{\partial z}{\partial x}\,,\ q = \frac{\partial z}{\partial y}\right)$$

(d) Define Cauchy problem for second order partial differential equations.

(e) Prove that for a particle moving in a central force field the areal velocity is constant.

(f) A particle moves in a plane with constant speed. Prove that its acceleration is perpendicular to its velocity.

(g) What do you mean by 'Constrained motion' of a particle? Give an example of such motion.

(h) Define central force field and give an example of non-central force.

2. Answer *any four* questions : 5×4=20

(a) Find the general integral of the PDE.

$$(x-y)y^2p + (y-x)x^2q = (x^2+y^2)z$$

and also find the particular integral passing through the curve $xz = 1,\ y = 0$. 3+2=5

(b) Reduce the second order PDE,

$$U_{xx} + 2U_{xy} + U_{yy} = 0 \text{ to Canonical form.}$$

State the nature of the PDE. 4+1=5

(c) Solve the initial value problem

$$\partial u/_{dx} + 2\,\partial u/_{dy} = 0\,, \qquad u(0,y) = 4e^{-2y}$$

by method of separation of variables. 5

(d) Solve the Cauchy problem of an infinite string with initial condition as given by

$$\partial^2 u/\partial t^2 = c^2\, \partial^2 u/\partial x^2 \,,\ x \in \mathbb{R},\ t > 0$$

$$U(x, 0) = f(x),\ x \in \mathbb{R}$$

$$\partial u/\partial t = 0 \text{ at } t = 0,\ x \in \mathbb{R}$$

by the method of characteristics. 5

(e) A particle describes an elliptic orbit under a central force which is always directed towards a focus of the orbit, find the law of force and the velocity at any point in the orbit. 5

(f) A particle falls down a cycloid under its own weight starting from the cusp. Show that when it arrives at the vertex the pressure on the curve is twice the weight of the particle. 5

3. Answer *any one* question : 10×1=10

(a) (i) Determine the d'Alembert solution of the Cauchy problem for one-dimensional homogeneous wave equation.

(ii) State the problem of vibration of semi-infinite string with a free end and give the solution by using d'Alembert's solution. 5+5=10

(b) (i) Show that for a particle moving in a plane curve under a conservative system of forces, the sum of its kinetic and potential energy is constant.

(ii) Show that the gravitational potential function $V = \mu/r$, where μ is a constant and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, $r = |\vec{r}|$ satisfies the Laplace equation. 5+5=10

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