B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52116 **Course Code : SHMTH-503-DSE-1**

Course Title: Linear Programming

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mention, symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Justify the statement that vertex is a boundary point, but all boundary points are not vertices.

(b) Write the dual of Minimize $6x_1 + 3x_2$, subject to $3x_1 + 4x_2 + x_3 \geq 5$; $6x_1 - 3x_2 + x_3 \geq 2$; $x_i \geq 0$, $i = 1, 2, 3$.

(c) When an LPP has multiple solution, answer in the context of simplex method.

(d) Determine the convex hull of the set $A = \{(x_1, x_2) ;\ x_1^2 + x_2^2 = 1\}$.

(e) Is $(2, 0, 0, 1, 0)$ a basic feasible solution of the system of equations

$2x_1 + 2x_2 - x_3 + x_4 + 5x_5 = 5$, $4x_1 + 3x_2 - 3x_3 + 2x_4 - 3x_5 = 10$? Justify.

(f) Write down the mathematical formulation of an assignment problem.

(g) For what value of λ the game with following pay-off matrix is strictly determinable?

	B_1	B_2	B_3
A_1	λ	7	5
A_2	–2	λ	–8
A_3	–3	4	λ

(h) In a transportation problem with 3 origins and 4 destinations, can the variables x_{11}, x_{12}, x_{22}, x_{34}, x_{23}, x_{13} be considered as basic variables? Give reason.

2. Answer *any four* questions: 5×4=20

(a) The first tableau of an L.P.P. by simplex method is given below (for a maximization problem) in an incomplete form:

C_j										
C_B	B	X_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7
0	a_5	x_5	20	– 4	6	5	– 4			
0	a_6	x_6	10	3	– 2	4	1			
0	a_7	x_7	20	8	– 3	3	2			
$Z_j - C_j$				– 4	– 1	– 3	– 5	0	0	0

(i) Complete the objective row and the tableau. 1

(ii) Write down the L.P.P. in its standard form. 1

(iii) Write down the actual problem. 1

(iv) Find the departing and the entering vector and write down the next tableau. 1+1=2

(b) By solving the dual of the following problem, show that the given problem has no feasible solution. 5

Minimize $Z = x_1 - x_2$

subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0.$

(c) Use Big-M method to solve the following L.P.P.: 5

Maximize $Z = 3x_1 - x_2$

subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0$.

(d) Solve the following assignment problem with the cost matrix: 5

	M_1	M_2	M_2	M_4	M_5
I	5	11	10	12	4
II	2	4	6	3	5
III	3	12	5	14	6
IV	6	14	4	11	7
V	7	9	8	12	5

and find the minimum cost of assisgnment,

(e) Find the optimal solution and the corresponding cost of transportation in the following transportation problem: 5

	D_1	D_2	D_3	D_4	a_i
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_i	5	8	7	14	

(f) Solve graphically the game whose pay-off matrix is

$$\begin{pmatrix} 2 & 2 & 3 & -1 \\ 4 & 3 & 2 & 6 \end{pmatrix}.$$

3. Answer *any one* questions: 10×1=10

(a) (i) Prove that there are only $(m+n-1)$ independent equations in a T.P., m, n being the number of origin and destinations and only one equation can be dropped as being redundant. 3

(ii) Show that set of all feasible solutions of an L.P.P. is convex set. 2

(iii) Using simplex method, obtain the inverse of the matrix $= \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$. 5

(b) (i) Prove that dual of the dual is the primal. 3

(ii) In a rectangular game the pay-off matrix A is given by

$$A = \begin{pmatrix} 3 & 2 & -1 \\ 4 & 0 & 5 \\ -1 & 3 & -2 \end{pmatrix}.$$

State giving reasons whether the payers will use pure or mixed strategies. What is the value of the game? 5

(iii) Solve the following assignment problem with the cost matrix

5	11	10	12	4
2	4	6	3	5
3	12	5	14	6
6	14	4	11	7
7	9	8	12	5

and find the minimum cost of Assignment. 2

B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52116 **Course Code : SHMTH-503-DSE-1**

Course Title: Theory of Equations

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mention, symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Find the value of a such that $x^4+3x^3+x^2+ax+3$ is exactly divisible by $x+1$.

(b) From a biquadratic equation with rational coefficients, two of whose roots are $2i \pm 1$.

(c) $x^3+3px+q$ has a factor of the form $(x-\alpha)^2$. Show that $q^2+4p^3=0$.

(d) State Descartes's rule of signs. Using this rule find the minimum number of complex roots of the equation $x^5-4x^3+3x-9=0$.

(e) Find an upper limit of the real roots of the equation $x^4-2x^3+3x^2-2x+2=0$.

(f) If α be a multiple root of order 3 of the equation $x^4+bx^2+cx+d=0$ $(d \neq 0)$, show that

$\alpha = -\frac{8d}{3c}$.

(g) Find the condition that the equation $x^3+px^2+qx+r=0$ may have two roots equal but of opposite signs.

(h) Find the maximum and minimum values of the polynomial, if exist

$f(x)=3x^4-16x^3+6x^2-48x+7$.

2. Answer *any four* questions: 5×4=20

(a) (i) If the equation $x^4+ax^3+bx^2+cx+d=0$ have three equal roots, prove that each of them is equal to

$\frac{6c-ab}{3a^2-8b}$.

(ii) If the roots of the equation $x^3+px^2+qx+r=0$ are in A.P., show that $p^2 \geq 3q$. 3+2=5

(b) (i) Show that the condition of sum of two roots of the equation $x^4+mx^2+nx+p=0$ be equal to the product of the other two roots is

$$(2p-n)^2=(p-n)(p+m-n)^2.$$

(ii) If α, β, γ be the roots of the equation $x^3 + qx + r = 0$, find the value of $\sum \frac{1}{\alpha^2 - \beta\gamma}$. 3+2=5

(c) Find the relation among the coefficients of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, if its roots $\alpha, \beta, \gamma, \delta$ be connected by the relation $\alpha + \beta = \gamma + \delta$. 5

(d) If α, β, γ be the roots of the equation $x^3 + 3x + 1 = 0$, find the equation whose roots are

$\frac{1}{\beta^2} + \frac{1}{\gamma^2} - \frac{1}{\alpha^2}, \frac{1}{\gamma^2} + \frac{1}{\alpha^2} - \frac{1}{\beta^2}, \frac{1}{\alpha^2} + \frac{1}{\beta^2} - \frac{1}{\gamma^2}$. 5

(e) Show that if the roots of the equation $x^4 + x^3 - 4x^2 - 3x + 3 = 0$ are increased by 2, the transformed equation is a reciprocal equation. Solve the reciprocal equation and hence obtain the solution of the given equation. 5

(f) Express $f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$ in the form $(x^2 - 5x + \lambda)^2 - (ax + b)^2$. Hence solve $f(x) = 0$. 5

3. Answer *any one* question: 10×1=10

(a) (i) Apply Sturm's theorem to find the number and position of the real roots of the equation

$x^4 - 6x^3 + 10x^2 - 6x + 1 = 0$.

(ii) Show that roots of the equation

$\frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3} = \frac{1}{x}$ are all real. 6+4=10

(b) (i) Prove that the equation $(1 + x)^5 = a(1 + x^5)$ is a reciprocal one, where $a \neq 1$. Solve it when $a = 5$.

(ii) Prove that the special roots of the equation $x^9 - 1 = 0$ are the roots of the equation $x^6 + x^3 + 1 = 0$ and show that the roots of the equation $x^6 + x^3 + 1 = 0$ are $\alpha, \alpha^2, \alpha^4, \alpha^5, \alpha^7, \alpha^8$ where $\alpha = e^{\frac{2\pi i}{9}}$. (2+3)+(2+3)=10

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B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52116 **Course Code : SHMTH-503-DSE-1**

Course Title: Point Set Topology

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mention, symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Find all compact subsets of a discrete metric space.

(b) Show that $\left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$ is compact in $\mathbb{R}$.

(c) Let A, B be compact subsets of a metric space X. Is $A \cap B$ compact?

(d) State Axiom of choice.

(e) Define a connected topological space.

(f) State Baire Category theorem.

(g) Let X have the discrete topology and Y is any topological space then show that any function $f : X \to Y$ is continuous.

(h) When is a finite subset of a metric space connected?

2. Answer *any four*: 5×4=20

(a) Prove that countable union of countable sets is countable. 5

(b) Let (X, d) be a metric space. Let us consider another metric $\bar{d}$ on X defined by $\bar{d}(x, y) = min\{1, d(x, y)\}$ for $x, y \in X$. Then prove that the metric topology on X induced by d is same as the metric topology on X induced by $\bar{d}$. 5

(c) (i) Let X be a discrete topological space. Determine the closure of any subset A of X. Also determine the dense subsets of X. 1+1

(ii) Let $f : X \to Y$ be any function. If $(Y,\ \tau_0)$ is an indiscrete space then show that

$f :\ (X, \tau) \to (Y, \tau_0)$ is continuous for any τ. 3

(d) (i) Show that the circle

$\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ is connected. 3

(ii) Is the space $\mathbb{R}^2 \backslash \{(0,0)\}$ connected? Justify. 2

(e) Show that continuous image of a compact metric space is compact.

(f) (i) What is the base of $\mathbb{R}$ endowed with the usual topology? 1

(ii) Write down a base for the discrete topology on X. 1

(iii) Show that all open intervals in $\mathbb{R}$ are homeomorphic. 3

3. Answer any one: 10×1=10

(a) (i) Let $X = \{a, b, c, d, e\}$ and let $A = \{\{a, b, c\}, \{c, d\}, \{d, e\}\}$. Find the topology on X generated by A. 3

(ii) Show that the usual topology U on the real line $\mathbb{R}$ is coarser than the upper limit topology T on $\mathbb{R}$ which has as a base the class of open-closed intervals $(a, b]$. 2

(iii) Let $f : X \to \mathbb{R}$ be a real continuous function defined on a connected set X. Then prove that f assumes as a value each member between any two of its values. 5

(b) (i) Let (X, τ) be a topological space and $\{A_\alpha\}$ be a family of connected subspaces of X. Prove that if $\bigcap_{\alpha \in A} A_\alpha \neq \phi$ then $\bigcup_{\alpha \in A} A_\alpha$ is connected. 5

(ii) Prove that every complete metric space is of second category. 5

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B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52117 **Course Code : SHMTH-504-DSE-2**

Course Title : Probability and Statistics

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* of the following: 2×5=10

(a) If A and B are any two events, then prove that the probability of occurring exactly one of them is given by $P(A) + P(B) - 2P(A \cap B)$.

(b) If the random variable X has the probability density $f(x) = \begin{cases} ke^{-3x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$

find k and $P(0{\cdot}5 \leq X \leq 1)$.

(c) Find the expectation of the random variable X where $X = \begin{cases} 1 & \text{if } A \text{ happens} \\ 0 & \text{if } A \text{ does not happen.} \end{cases}$

(d) If the random variable X and Y have the same standard deviation, show that $U = X + Y$ and $V = X - Y$ are uncorrelated.

(e) Show that the correlation coefficient ρ satisfies the inequality $-1 \leq \rho \leq 1$.

(f) How that the sample mean is unbiased estimator of population mean.

(g) Show that the distribution function $F(x)$ is monotonic non-decreasing on R.

(h) Define sample characteristic. Write expression for sample variance and sample k-th central moment.

2. Answer *any four* questions from the following: 5×4=20

(a) If the joint probability density of X and Y is given by

$$f(x, y) = \begin{cases} 2 & \text{for } x > 0, y > 0, x + y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

find (i) $P\left(X \leq \frac{1}{2}, Y \leq \frac{1}{2}\right)$

(ii) $P\left(X + Y > \frac{2}{3}\right)$

(iii) $P(X > 2Y)$

(b) State and prove Chebyshev's inequality for a continuous random variable.

(c) If X and Y are independent Poisson variates, show that the conditional distribution of X given $X + Y$ is binomial.

(d) Define normal distribution and find mean and variance of a normal random variate.

(e) If $\overline{X}$ be the sample mean of a random sample $(X_1, X_2, \cdots, X_n)$ drawn from an infinite population with mean μ and variance σ^2 then show that

(i) $E(\overline{X}) = \mu$

(ii) $\text{var}\,(\overline{X}) = \sigma^2/n$, and

(iii) $E\left(\frac{n}{n-1}\,S^2\right) = \sigma^2$ where

S^2 is the sample central moment of order 2.

(f) Obtain the maximum likelihood estimate of θ on the basis of a random sample of size n drawn from the population whose p.d.f. is given by $f(x) = c\,e^{-x/\theta}, 0 \le x < \infty$, where c is a constant and $\theta > 0$. Discuss consistency and un-biasedness of the estimate.

3. Answer *any one* question from the following: 10×1=10

(a) (i) Find the mean and variance of a Poisson random variable with parameter m.

(ii) State central limit theorem for independent and identically distributed random variables with finite variance.

(iii) Consider a Markov chain with state space $\{0, 1, 2, 3\}$ and transition probability matrix

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Determine which states are transient and which are recurrent. 5+2+3=10

(b) (i) The marks obtained by 18 candidates in an examination having a mean 56 and variance 65. Find 95% confidence interval for the mean of the population of marks, assuming it to be normal.

[For 17 degrees of freedom $P(|t| > 2{\cdot}11) = 0{\cdot}05$]

(ii) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test. 5+5=10

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B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52117 **Course Code : SHMTH-504-DSE-2**

Course Title : Boolean Algebra and Automata

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* of the following: 2×5=10

(a) State the Pumping Lemma for regular languages.

(b) Briefly prove or disprove: Every finite lattice is complete.

(c) Briefly prove or disprove: Homomorphic image of a distributive lattice is distributive.

(d) Define a Turing machine.

(e) Is there a unique DFA corresponding to any state diagram realizing it? If yes, prove it; if no, give example.

(f) An *antichain* is a subset of a partially ordered set such that any two distinct elements in the subset are incomparable.

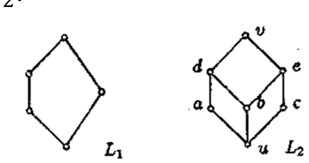
Given an example of an ordered set P such that $|P| > 3$ in which there are three elements x, y, z such that

i) $\{x, y, z\}$ is an antichain.

ii) $x \vee y$, $y \vee z$ and $z \vee x$ fail to exist.

iii) $\vee\{x, y, z\}$ exists.

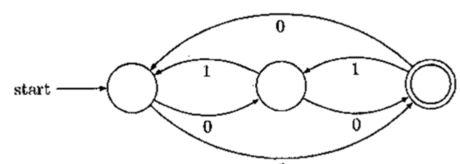
(g) Find L_1 as a sublattice of L_2.



(h) Let $\sum = \{1\}$. Prove that there is an undecidable subset of $\sum^*$.

2. Answer *any four*: 5×4=20

(a) Convert the following DFA to a regular expression.



(b) Let $\sum = \{a, b\}$. Let L be a language over $\sum$, consisting of strings of length at least 2, where the first letter is the same as the last letter, and the second letter is the same as the second to last letter. For example, $a \notin L$, $b \notin L$, $aa \in L$, $aaa \in L$, $aba \in L$, $bbaabba \notin L$. Design a DFA that accepts L.

(c) Draw the state diagram of a Pushdown Automata realizing $\{0^n 1^n | n \geq 0\}$.

(d) Define Kleene closure of a regular language A, as $A^* = \{x_1, x_2 \ldots x_k | k \geq 0$ and each $x_i \in A\}$. Prove that Kleene closure of every regular language is regular.

(e) Let P and Q be finite ordered sets and let $\psi : P \to Q$ be a bijective map. Then prove the following to be equivalent:

(i) $x < y$ in P if and only if $\psi\,(x) < \psi\,(y)$ in Q.

(ii) $x \sim < y$ in P if and only if $\psi\,(x) \sim < \psi\,(y)$ in Q.

(f) Prove that any distributive lattice is modular. Is the converse true? Justify. 3+2=5

3. Answer *any one*: 10×1=10

(a) (i) Define, $A_{DFA} = \{< B, \omega > | B$ is a DFA that accepts input string $\omega\}$. Prove that, A_{DFA} is a decidable language.

(ii) Prove that, every nondeterministic Turing machine has an equivalent deterministic Turing machine. 5+5=10

(b) (i) Let $f : B \to C$ where B and C are Boolean algebras.

A. Assume f to be a lattice homomorphism. Then prove the following to be equivalent:

$f(0) = 0$ and $f(1) = 1$.

$f(a') = (f(a))'$, $\forall a \in B$.

B. Also prove that, if f preserves $'$, then f preserves $\vee$ if and only if f preserves $\wedge$.

(ii) Let L and K be lattices and $f : L \to K$ a map. Prove the following to be equivalent:

A. f is order preserving.

B. $(\forall a, b \in L)\ f(a \vee b) \geq f(a) \vee\ f(b)$.

C. $(\forall a, b \in L)\ f(a \vee b) \leq f(a) \wedge\ f(b)$. 5+5=10

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SP-V/Math-501-DSE-1A/19

B.Sc. 5th Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 52118 **Course Code : SPMTH-501-DSE-1A**

Course Title : Theory of Equations

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* of the following: 2×5=10

(a) Find the quotient and remainder when $x^4 + 5x^3 + 4x^2 + 8x - 2$ is divided by $x + 2$.

(b) Apply Decarte's rule of signs to find the nature of the roots of the equation $x^4 + 2x^2 + 3x - 1 = 0$.

(c) Prove that $x^{40} + x^{23} + x^{30} + x^{13}$ is divisible by $x^2 + 1$.

(d) If α, β, γ be the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, find the value of $\Sigma\, \alpha^2\beta$.

(e) If a, b are the roots of the equation $x^2 - px + q = 0$, then find the equation whose roots are $\frac{1}{a}, \frac{1}{b}$.

(f) Form a cubic equation whose roots are 2, $3 - 2i$.

(g) Verify that $(x + 1)^4 + x^4 + 1 = 0$ is a reciprocal equation.

(h) If α be an imaginary root of the equation $x^n - 1 = 0$, where n is a prime number, prove that $(1 - \alpha)(1 - \alpha^2) \cdots (1 - \alpha^{n-1}) = n$.

2. Answer *any four* questions from the following: 5×4=20

(a) State the fundamental theorem of classical algebra. If the equation $x^4 + px^2 + qx + r = 0$ has three equal roots then show that $8p^3 + 27q^2 = 0$ and $p^2 + 12r = 0$.

(b) If α, β, γ be the roots of the equation $x^3 + 3x + 1 = 0$, find an equation whose roots are $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}, \frac{\beta}{\gamma} + \frac{\gamma}{\beta}, \frac{\gamma}{\alpha} + \frac{\alpha}{\gamma}$.

(c) If α be a root of the equation $x^3 - 3x - 1 = 0$, prove that the other roots are $2 - \alpha^2, \alpha^2 - \alpha - 2$.

(d) If α, β, γ are the roots of the equation $x^3 + 3x^2 + 10 = 0$, prove that $\alpha^4 + \beta^4 + \gamma^4 = 201$.

(e) If α is a special root of the equation $x^8 - 1 = 0$, then show that $(\alpha + 2)(\alpha^2 + 2) \cdots (\alpha^7 + 2) = 85$.

(f) Reduce the equation, $6x^6 + 25x^5 + 31x^4 - 31x^2 - 25x - 6 = 0$ to a reciprocal equation of the standard form and then solve it.

3. Answer *any one* question from the following: 10×1=10

(a) (i) Use Sturm's theorem to show that $x^3 - 7x + 7 = 0$ has two roots between 1 and 2 and the other root between – 3 and – 4.

(ii) Solve by Cardan's method, the equation $x^3 - 6x^2 - 6x - 7 = 0$. 5+5=10

(b) (i) Prove that $f(x) = 0$ be a reciprocal equation of degree *n* and of the first type if and only if $f(x) = x^n\, f\left(\frac{1}{x}\right)$.

(ii) If α be an imaginary root of the equation $x^7 - 1 = 0$, find the equation whose roots are $\alpha + \alpha^6, \alpha^2 + \alpha^5, \alpha^3 + \alpha^4$. 5+5=10

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B.Sc. 5th Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 52118 **Course Code : SPMTH-501-DSE-1A**

Course Title : Linear Programming

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* of the following: 2×5=10

(a) Define Basic feasible solution and optimal solution.

(b) Examine if $X = \{(x, y) : |X| \leq 2\}$ is a convex set.

(c) Prove that $x_1 = 2, x_2 = 3, x_3 = 0$ is a feasible solution but not basic solution of the set of equations $3x_1 + 5x_2 - 7x_3 = 21$; $6x_1 + 10x_2 + 3x_3 = 42$.

(d) Apply maximum and minimum principle to solve the game whose pay-off matrices are

$$\begin{pmatrix} 15 & 2 & 3 \\ 6 & 5 & 7 \\ -7 & 4 & 0 \end{pmatrix}$$

(e) What is the condition for optimality and entering variable in simplex table?

(f) What is two-person zero sum game? Give a suitable example.

(g) Write down the mathematical formulation of an assignment problem.

(h) State fundamental theorem of duality.

2. Answer *any four* questions from the following: 5×4=20

(a) Food X contains 6 units of vitamin A and 7 units of vitamin B per gram and costs 12P./gm. Food Y contains 8 units and 12 units of A and B per gram respectively and costs 20 p./gm. The daily requirement of vitamin A and B are at least 100 units and 120 units respectively. Formulate the above as an L.P.P. to minimize the cost. Then solve it graphically.

(b) Use duality to solve the following L.P.P.

Minimize $Z = 3x_1 + x_2$

Subject to $2x_1 + 3x_2 \geq 2$

$x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0.$

(c) Solve the following game graphically.

		Player B		
		B_1	B_2	B_3
Player-A	A_1	1	3	11
	A_2	8	5	2

(d) Define convex set. Show that the set of all feasible solution of a L.P.P. is a convex set. 1+4=5

(e) Solve the following L.P.P. graphically:

Maximize $Z = -x + 2y$.

Subject to $-x + y \leq 1; -x + 2y \leq 4; x, y \geq 0$.

(f) Find the optimal solution on the transportation problem

2	2	2	1	3
10	8	5	4	7
7	6	6	8	5
4	3	4	4	

3. Answer *any one* question: 10×1=10

(a) (i) $x_1 = 1\ x_2 = 2, x_3 = 1$ and $x_4 = 0$ is a feasible solution to the set

$11x_1 + 2x_2 - 9x_3 + 4x_4 = 6$

$15x_1 + 3x_2 - 12x_3 + 5x_4 = 9$

Reduce F.S. to one B.F.S.

(ii) Solve the following transportation problem by Matrix minima method.

2	2	3	10
4	1	2	15
1	3	1	40
20	15	30	

(iii) Define saddle point. 4+4+2=10

(b) (i) Solve the following L.P.P. by using two phase simplex method

Minimize $Z = x_1 - 2x_2 - 3x_3$

Subject to $-2x_1 + x_2 + 3x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 1,\ \ x_1, x_2, x_3 \geq 0$

(ii) If we add a fixed number to each element of a pay-off matrix, the optimal strategies remain unchanged but the value of the game is increased by that number. 5+5=10

SP-V/Math-504-SEC-3/19

B.Sc. 5th Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 52110 **Course Code : SPMTH-504-SEC-3**

Course Title : Programming Using C

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meaning.

1. Answer *any five* of the following: 2×5=10

(a) Write down two differences between compiler and interpreter.

(b) Express the following expression as valid C expression: $x^3 + \sqrt{x} + \sin x + e^{x^2+x+1}$.

(c) Write a short note on relational operators in C.

(d) Which of the following are invalid variables and why?
2019_PDAY, Christian_College, # 420, INTEGER

(e) Find the difference between = and = = symbol in C programming.

(f) What will be the output of $4 + 2\% - 8$?

(g) What do you mean by $!\,(y < 10)$?

(h) Find the output of the following:

```
int i=1; while (i<=10)
            {
             printf ("%d\n",i);
             i=i++
            }
```

2. Answer *any four* questions from the following: 5×4=20

(a) What is the machine language? State its difficulties and explain how we can get rid of these difficulties. 1+2+2=5

(b) Write a complete c program to determine the area of a triangle using the formula:
Area $= \sqrt{s\,(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$, and a, b and c are the sides of the triangle. 5

(c) Define function subprograms in C, with a suitable example. Explain the use of 'return' statement in connection with the functions. 2+1+2=5

(d) Abhijit's basic salary is input through keyboard. His DA is 30% of basic salary and HRA is 12% of basic salary. Write a program to calculate his gross salary. 5

(e) If a three digit number is input through keyboard, write a program to obtain the sum of the first and last digit of this number.

(f) (i) What will be the output of the following:

```
int x=3, y=5; if(x==3)
             printf ("%d\n",x);
             else;
             printf("%d\n",y);
```

(ii) Define source and object programs. 2+3=5

3. Answer *any one* question from the following: 10×1=10

(a) (i) What will be the output of the following:

```
printf ("aa\n\n aaa\n");

printf("aaa/n/n a");
```

(ii) Find the value of $!(a > 5 \,\&\&\, c)$

(iii) Write a program to find factorial of a number. 2+2+6=10

(b) (i) Write short notes on:

(I) Assignment operators

(II) Logical operators

(ii) Write a C program to find the sum up to *n* terms of the series: $1 + x + x^2 + x^3 + \cdots$.

(3+3)+4=10

B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52111 **Course Code : SHMTH-501-C-11**

Course Title: Partial Differential Equation and Applications

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer *any five* questions : 2×5=10

(a) When a first order PDE is said to be Linear? Give an example of it.

(b) Form a partial differential equation by diminating constants A and p from $Z = Ae^{pt}\sin px$.

(c) Show that all the surfaces of revolution of the form $Z = f(x^2 + y^2)$ with the Z-axis as the axis of symmetry, where f is an arbitrary function, satisfy the PDE.

$$yp - xq = 0, \quad \left(p = \frac{\partial z}{\partial x}\ ,\ q = \frac{\partial z}{\partial y}\right)$$

(d) Define Cauchy problem for second order partial differential equations.

(e) Prove that for a particle moving in a central force field the areal velocity is constant.

(f) A particle moves in a plane with constant speed. Prove that its acceleration is perpendicular to its velocity.

(g) What do you mean by 'Constrained motion' of a particle? Give an example of such motion.

(h) Define central force field and give an example of non-central force.

2. Answer *any four* questions : 5×4=20

(a) Find the general integral of the PDE.

$$(x-y)y^2p + (y-x)x^2q = (x^2+y^2)z$$

and also find the particular integral passing through the curve $xz = 1,\ y = 0$. 3+2=5

(b) Reduce the second order PDE,

$$U_{xx} + 2U_{xy} + U_{yy} = 0 \text{ to Canonical form.}$$

State the nature of the PDE. 4+1=5

(c) Solve the initial value problem

$$\partial u/_{dx} + 2\,\partial u/_{dy} = 0\,, \qquad u(0,y) = 4e^{-2y}$$

by method of separation of variables. 5

(d) Solve the Cauchy problem of an infinite string with initial condition as given by

$\partial^2 u/\partial t^2 = c^2\, \partial^2 u/\partial x^2$, $x \in \mathbb{R},\ t > 0$

$U(x, 0) = f(x),\ x \in \mathbb{R}$

$\partial u/\partial t = 0$ at $t = 0,\ x \in \mathbb{R}$

by the method of characteristics. 5

(e) A particle describes an elliptic orbit under a central force which is always directed towards a focus of the orbit, find the law of force and the velocity at any point in the orbit. 5

(f) A particle falls down a cycloid under its own weight starting from the cusp. Show that when it arrives at the vertex the pressure on the curve is twice the weight of the particle. 5

3. Answer *any one* question : 10×1=10

(a) (i) Determine the d'Alembert solution of the Cauchy problem for one-dimensional homogeneous wave equation.

(ii) State the problem of vibration of semi-infinite string with a free end and give the solution by using d'Alembert's solution. 5+5=10

(b) (i) Show that for a particle moving in a plane curve under a conservative system of forces, the sum of its kinetic and potential energy is constant.

(ii) Show that the gravitational potential function $V = \mu/r$, where μ is a constant and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\vec{r}|$ satisfies the Laplace equation. 5+5=10

B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52112 **Course Code : SHMTH-502-C-12**

Course Title: Group Theory - II

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer *any five* questions from the following: 2×5=10

(a) Define inner automorphism on a group.

(b) Show that every element of a commutator subgroup G' of a group G is a product of square.

(c) Is the group $\mathbb{Z} \times \mathbb{Z}$ cyclic? Justify your answer.

(d) Show that $Z(G)$, the centre of the group G, is a characteristic subgroup of G.

(e) Write down the class-equation of S_3 with justification.

(f) Let G be a group of order 15. Prove that G is cyclic.

(g) Give example of an infinite p-group where p is a prime.

(h) State Sylow's Third Theorem.

2. Answer *any four* questions from the following: 5×4=20

(a) Let G be a group. Then show that $G/Z(G)$ is isomorphic to $Inn(G)$ where $Inn(G)$ denotes the group of all inner automorphisms of G. 5

(b) (i) Define characteristic subgroup of a group.

(ii) Prove that every characteristic subgroup of G is normal in G.

(iii) Give example of a group G and a subgroup H of G such that H is normal in G but H is not a characteristic subgroup of G. 1+2+2=5

(c) Describe all abclian group of order 360. 5

(d) (i) Let G be a finite group and H be a proper subgroup of G of index n such that $|G| \nmid n!$. Then show that G contains a non-trivial normal subgroup.

(ii) Let G be a group of order 65 and H be a subgroup of order 13. Prove that G is not simple. 3+2=5

(e) Let G be a group of order p^2 where p is a prime. First show that $Z(G)$ non-trivial. Then prove that G is abelian. 2+3=5

(f) Use Sylow's Theorems to show that any group of order 36 is not simple. 5

3. Answer *any one* question from the following: 10×1=10

(a) (i) Prove that $Aut(\mathbb{Z}_8)$ is isomorphic to the Klein's Four Group. 2

(ii) Prove that the group $A \times B$ is abelian if and only if both of the groups A & B are abelian. 2

(iii) Let G be a group acting on a non-empty set S. Prove that the index of G_a, the stabilizer subgroup of a $(a \in s)$, in G is equal to the cardinality of the orbit of a. 4

(iv) Prove that any 5-Sylow subgroup of a group of order 45 is always normal. 2

(b) (i) Find all automorphisms of the group $(\mathbb{Z}_6\,,+)$. 2

(ii) Is there any element of order 12 in the additive group $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_3$. 3

(iii) Let G be a group of order $105 = 3{\cdot}5{\cdot}7$. First show that any Sylow 7-subgroup and any Sylow 5-subgroup are normal in G. Then show that G has a cyclic subgroup of order 35. Finally show that if H is Sylow 7-subgroup, K is a Sylow 5-subgroup and L is a Sylow 3-subgroup of G then $G = HKL$. 1+1+1+2=5

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B.Sc. 5th Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 52112 **Course Code : SHMTH-502-C-12**

Course Title: Group Theory - II

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer *any five* questions: 2×5=10

(a) Prove that $Aut\ \mathbb{Z}_{10} \cong \mathbb{Z}_4$.

(b) Give an example of a subgroup which is not characteristic. Justify your answer.

(c) Show that the direct product $\mathbb{Z} \times \mathbb{Z}$ is not a cyclic group.

(d) How many elements of order 7 are there in a simple group of order 168?

(e) Prove that every group of order 15 is cyclic.

(f) Let G be a finite group that has only two conjugatie classes. Show that $|G| = 2$.

(g) Show that every group of order 45 has a normal subgroup of order 9.

(h) Prove that $Inn\ S_3 \cong S_3$.

2. Answer *any four* questions: 5×4=20

(a) (i) Show that for any group G, ${}^{G}/_{Z(G)} \cong Inn\ G$. 3

(ii) Find $Inn\ (D_4)$. 2

(b) Let G be a finite group. Let H be a subgroup of G of index p, where p is the smallest prime dividing $|G|$. Show that H is a normal subgroup of G.

(c) Define internal direct product of finite number of subgroups and show that if $G$is the internal direct product of its subgroups $H_1,\ H_2 \ldots \ldots \ldots ,\ H_n$; then $G \cong H_1 \times H_2 \times \ldots \ldots \ldots ,\ H_n$. 1+4=5

(d) Let G be a finite p-group with $|G| > 1$. Prove that $|Z(G)| > 1$, where $Z(G)$ is the centre of G.

(e) (i) Define Sylo p-subgroup.

(ii) Prove that no group of order 56 is simple. 1+4=5

(f) (i) Define group action with example. 1+1=2

(ii) Let G be a finite group and S be a finite G-set. Then prove that the number of orbits of S is $\frac{1}{|G|}\sum_{g\in G} F(g)$, where $F(g)$ is the number of elements of S fixed by g. 3

3. Answer *any one* question: 10×1=10

(a) (i) Let G be a group of order $2m$, where m is an odd integer. Show that G has a normal subgroup of order m. 4

(ii) Let G be a group and S be a G-set, then prove that for all $a \in s$, $[G\colon G_a] = |[a]|$. 3

(iii) Let H be a subgroup of a group G. Prove that if H has a finite index n, then there is a normal subgroup K of G with $K \subseteq H$ and $[G\colon K] \leq n!$. 3

(b) (i) Prove that $|G| = |Z(G)| + \sum_{a \notin Z(G)}[G\colon C(a)]$, where the summation runs over the complete set of distinct conjugacy class representative which does not belong to $Z(G)$. 3

(ii) State and prove Sylow's Second Theorem. 1+3=4

(iii) Let G be a finite group of order p^n where p is a prime and $n \geq 1$. Prove that any subgroup of G of order $p^{n-1}$is a normal subgroup of G. 3