B.SC. FIFTH SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 52111**

Course Code: SH/MTH/501/C-11

Course Title: Partial Differential Equations and Applications

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

1. **Answer *any five* of the following questions:** $(5\times 2 = 10)$

   a) Give an example of Initial-Boundary Value Problem (IBVP) in PDE and indicate the initial and boundary conditions.

   b) When a first order partial differential equation (PDE) is said to be *quasi-linear*? Give an example of it.

   c) Solve: $(y-z)p + (z-x)q = x - y$.

   d) For what value(s) of k the equation $u_{xx} - 2u_{xy} + k\,u_{yy} - 3u_y = 0$ represents an elliptic PDE?

   e) Form the PDE by eliminating the arbitrary functions from the relation
   $$z = xf(x+y) + g(x+y).$$

   f) Prove that if a particle moves in a central force field, then its path must be a plane curve.

   g) Find the characteristic equation of the following PDE:
   $$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0.$$

   h) State Kepler's laws of planetary motion.

2. Answer any *four* questions. $5\times 4 = 20$

a) Find the general integral of the PDE
$$(y+zx)p - (x+yz)q = x^2 - y^2$$
and also find the particular integral which passes through the line $x = 1,\ y = 0$.

3+2

b) Using the method of characteristic, find the solution of equation
$$u(x+y)u_x + u(x-y)u_y = x^2 + y^2$$
with the Cauchy data $u = 0$ on $y = 2x$.

c) Determine the solution of the following wave equation

$$u_{tt} = 4u_{xx}, \quad -\infty < x < \infty, \quad t > 0$$
$$u(x,0) = \sin x \quad -\infty < x < \infty$$
$$u_t(x,0) = x^2, \quad -\infty < x < \infty$$

by using d'Alembert's solution.

d) Define Central force and verify whether the principle of conservation of energy holds or not for motions under central force.

e) By using the method of separation of variables show that the solution of the equation $\frac{\partial^2\theta}{\partial x^2} = \frac{\partial\theta}{\partial t}$ satisfying the conditions (i) $\theta \to 0$ as $t \to \infty$, (ii) $\theta \to 0$ when $x = \pm l$ for all values of $t > 0, -l < x < l$ is

$\theta = \frac{2l}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\sin\frac{n\pi x}{l}e^{\left(-\frac{n^2\pi^2 t}{l^2}\right)}$.

f) Reduce the equation $u_x + xu_y = y$ to its canonical form and find the general solution.

3. Answer any *one* question. **$1 \times 10 = 10$**

a) i) Determine the solution of the following initial-boundary problem

$$u_{tt} - c^2u_{xx} = 0, \quad 0 < x < \infty, \quad t > 0$$
$$u(x,0) = f(x), \quad 0 \le x < \infty$$
$$u_t(x,0) = g(x), \quad 0 \le x < \infty$$
$$u(0,t) = 00 \le t < \infty$$

ii) A particle describes the curve $r^n = a^n \cos n\theta$ under a force to the pole. Find the law of force. 7+ 3

b) (i) Solve the linear equation $yu_x + xu_y = u$, with Cauchy data $u(x,0) = x^3$ and $u(0,y) = y^3$.

(ii) A uniform chain of length l and mass ml is coiled on a horizontal table, and a mass $M$is attached to one end of it. If this mass be projected vertically upwards, show that the least velocity of projection, so that whole chain may leave the table is

$$\frac{1}{m}\sqrt{\frac{2g}{3m}\{(M+ml)^3 - M^3\}}$$

where the mass per unit length is m. 6+4

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