POSTGRADUATE FOURTH SEMESTER EXAMINATIONS, 2022

Subject: Mathematics **Course ID: 42154**
Course Code: Math-404ME
Course Title: Computational Fluid Dynamics
Full Marks: 40 Time: 2 Hours

The figures in the margin indicate full marks

The notations and symbols have usual meanings unless stated otherwise

Answer *any five* of the following questions: **(8×5=40)**

1. **a)** What are the popular methods of studying fluid flow problems?

b)Which one of those methods is advantageous and why?

c) What are the differences (mention at least two) between a parabolic and a hyperbolic partial differential equation? Why is it more difficult to obtain computational solution of hyperbolic PDEs?

d) What is the meaning of pre-shock oscillation and why does it occur? (1+2+1+2+2)

2.

a) Discuss in brief the basic differences between the three discretization methods, namely, finite difference, finite volume and finite element methods. 3

b) Apply any one of the above mentioned methods to solve the Laplace equation (in two dimension, 2D) in a unit square with Dirichlet boundary condition (say equal to 1). Write down the scheme for a uniform grid. 5

3.

a) Derive the conservative form of the incompressible Navier-Stokes equations in 2D. 2

b) Define the staggered grid and show the locations of pressure and velocity components on a staggered grid in 2D. (1+1)

c) Using the staggered locations of the dependent variables discretize the unsteady, convective, diffusive and pressure gradient terms of the x-momentum equation in conservative form of incompressible Navier-Stokes equations in 2D. 4

4.

a) Use similar discretization process (as in 3(c)) to discretize the continuity and the y-momentum equations as well. 4

b) Use those discrete equations to generate an explicit algorithm to solve the system of unsteady Navier-Stokes equations. 4

5.

a) What do you understand when one talks about pseudo-transient formulation? What kind of flows can be computed using this formulation? (1+1)

b) Consider an iterative solution technique for solving 2D steady state Navier-Stokes equations using SIMPLE formulations:

(i) Draw the control volumes used in this formulation for the continuity and x-momentum as well as y-momentum equations. 2

(ii) Write down the discretized continuity and momentum equations used in SIMPLE-Algorithm. 4

6.

a) Define a Cauchy problem for scalar conservation law. Give an example of Cauchy problem.

b) Prove that the function $u \in L^{\infty}_{loc}(R_+ \times R)$ will be a weak solution of $u_t + f(u)_x = 0$, with $u(x,0) = u_0(x)$, if the equation $\int_{-\infty}^{\infty}\int_0^{\infty}(u\varphi_t + f(u)\varphi_x)\, dtdx + \int_{-\infty}^{\infty} u_0\,(x)\varphi(x,0)dx = 0$ is fulfilled for all test functions $\varphi \in C_0^1(R_+ \times R)$.

c) Define the Rankine-Hugoniot condition for a discontinuous solution.

d) Define the entropy solution. (2+3+2+1)

7. Consider the scalar conservation law $u_t + f(u)_x = 0$, with $u(x,0) = u_0(x), x \in R$.

a. Derive the Naive and the Lax-Friedrichs schemes for this problem.

b. Discuss in detail whether the schemes are conservative and consistent. (2+3+3)

8. Consider the linear advection equation $u_t + au_x = 0$, (with a being constant).

a. Compute the numerical viscosity $K\beta$ of the Naive scheme for this equation and conclude the behavior of the scheme based on $K\beta$.

b. In case of Lax-Friedrichs scheme, show that $K\beta$ is positive as long as $|a\lambda| < 1$.

(4+4)

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