

POSTGRADUATE FOURTH SEMESTER EXAMINATIONS, 2022

Subject: Mathematics

Course ID: 42153

Course Code: Math-403ME

Course Title: **Modelling and Analysis of Biological systems**

Full Marks: 40

Time: 2 Hours

The figures in the margin indicate full marks

The notations and symbols have usual meanings unless stated otherwise

Answer *any five* of the following questions:

(8×5=40)

1. (a) Write down a single species harvesting model.
(b) Find the maximum sustainable yield and the optimal harvesting rate.
(c) Explore transcritical bifurcation if exists.

$$2 + 3 + 3 = 8$$

2. (a) What is insect outbreak? Write an insect outbreak model.
(b) Discuss about the occurrence of saddle node bifurcation in the above-mentioned model.
(c) Also discuss the bistability if occurs in the above system.

$$(1 + 2) + 3 + 2 = 8$$

3. (a) Find the condition(s) for stability of the axial equilibrium point of the modified Lotka-Volterra model. Also provide biological interpretation of the above condition(s).
(b) Investigate the global stability of the interior equilibrium point.
(c) Write down two major differences between Lotka-Volterra model and modified Lotka-Volterra model.

$$(2 + 1) + 3 + 2 = 8$$

4. (a) Write down the Rosenzweig-MacArthur model.
(b) Find all the equilibrium points and investigate the local stability of the interior equilibrium point.
(c) Also discuss about the occurrence of Hopf bifurcation for gradual change in consumption rate.

$$2 + (2 + 2) + 2 = 8$$

5. (a) Write down the assumptions and sketch the schematic diagram for SIS (Susceptible-Infected-Susceptible) epidemic model.
(b) Using the next generation matrix find the basic reproduction number (R_0).

(c) Find the condition(s) for local stability of the disease-free equilibrium point.

$$3 + 3 + 2 = 8$$

6. (a) Write down a two-prey & one-predator model. Investigate the local stability of the axial and planar equilibrium points.

(b) Find the conditions for persistence of all species.

$$(2 + 4) + 2 = 8$$

7. (a) Write an intraguild predation model. Also discuss the main differences between simple food chain model and intraguild predation model.

(b) Find the conditions for stability of the planar equilibrium points of the above-mentioned model.

$$(2 + 2) + 4 = 8$$

8. (a) Write down the biological assumptions, draw the schematic diagram, and formulate an SEIR model for capturing the behavior of Covid 19 outbreak with partial lockdown.

(b) Investigate the local stability of the disease-free equilibrium point.

$$(2 + 1 + 2) + 3 = 8$$
