M.SC. FOURTH SEMESTER EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 42151**

Course Code: **Math-401C** **Course Title: Operations Research**

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

Answer any five from the following questions. $8 \times 5 = 40$

1. A manufacturer produces two types of products $P_1$and P_2 . Each P_1 product requires 4 hours of grinding and 2 hours of polishing, whereas each P_2 product requires 2 hours of grinding and 5 hours of polishing. The manufacture has 2 grinders and 3 polishers. Each grinder works for 40 hours a week, and each polisher works for 60 hours a week. Profit on P_1 product is Rs. 3 and on P_2 product is Rs. 4. Whatever is produced in a week is sold in the market.
   a) Write down the LPP to maximize the profit of the firm.
   b) Find optimal solution of the above LPP using simplex method. $4 + 4 = 8$
2. a) Convert the following LPP (primal) into its dual form

   Max $z = 3x_1 + 4x_2$

   Subject to the constraints:

   $2x_1 + 6x_2 \leq 16$

   $5x_1 + 2x_2 \geq 20$

   $\& \; x_1, x_2 \geq 0.$

   b) Solve the dual problem so obtained using dual simplex method. $2 + 6 = 8$

3. The optimal solution (table) for the following LPP:

   Maximize $z = 3x_1 + 2x_2$

   Subject to the constraints:

   $x_1 + x_2 \leq 4$

   $x_1 - x_2 \leq 2$

   and $x_1, x_2 \geq 0$

   is given below

c_j			3	2	0	0	Min $\left\{\frac{X_B}{x_i}, x_i > 0\right\}$
BV	C_B	X_B	x_1	x_2	s_1	s_2	
x_2	2	1	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	
x_1	3	3	1	0	$\frac{1}{2}$	$\frac{1}{2}$	
$\left(\sum C_{B_i} x_i\right) = z_j$			3	2	$\frac{5}{2}$	$\frac{1}{2}$	
$c_j - z_j$			0	0	$-\frac{5}{2}$	$-\frac{1}{2}$	

a) Using sensitivity analysis, find how much c_1 can be increased such that the optimality of the feasible solution is not disturbed?

b) Discuss the effect on the optimal solution for the change in the availability of resources from $[4\ \ 2]^T\ \ to\ \ [9\ 6]^T$. $4 + 4$

4. a) Solve the travelling salesman problem with the following data:

$$C_{12} = 15, C_{13} = 20, C_{14} = 10,$$
$$C_{21} = 15, C_{23} = 35, C_{24} = 15,$$
$$C_{31} = 20,\ C_{32} = 35, C_{34} = 32,$$
$$C_{41} = 10, C_{42} = 15, C_{43} = 32,$$

where C_{ij} is the cost of travelling from city 'i' to 'j'.

b) A manufacturing factory has 5 machines and 5 jobs to be performed. The time that each machine takes to perform each job is given in the following effectiveness matrix:

	J_1	J_2	J_3	J_4	J_5
M_1	40	35	25	20	15
M_2	16	26	48	50	24
M_3	22	12	32	15	17
M_4	16	18	12	15	36
M_5	45	20	36	24	18

Find the optimal job assignment and the optimal cost for completing all jobs. $4 + 4 = 8$

5. Using Gomory's mixed integer method, solve the following LPP:

   Max $z = x_1 + x_2$

   subject to the constraints:

   $2x_1 + 5x_2 \leq 16$

   $6x_1 + 5x_2 \leq 30$

   $x_2 \geq 0$ and x_1 is a non-negative integer. 8

6. Four jobs J_1,J_2,J_3 and J_4 are to be processed on each of the 4 machines A, B, C, D in the order ABCD. The processing times (in hours) are given as

Job i / Machine	J_1	J_2	J_3	J_4
A	11	13	9	16
B	4	3	5	2
C	6	7	5	8
D	15	8	13	9

   (a) Convert the above n-jobs and m-machines problem into n-jobs and 2-machines problem and find the optimal job sequence.

   (b) Find the total minimum elapsed time. Also find the idle time for each machine.

   $4 + 4 = 8$

7. Consider the following table summarizing the details of a project:

Activity	t_o	t_m	t_p
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(a) Find the expected duration and variation of each activity and construct the project network diagram.

(b) Find the critical path and expected project completion time. Also find the probability of completing the project on or before 20 weeks. 4+4=8

8. (a) What is inventory? Discuss the classification of inventories.

(b) Consider that a textile mill buys its raw material from a vendor. The annual demand of the raw material is 9000 units. The ordering cost is Rs. 100 per order and the carrying cost is 20% of the purchase price per unit per month, where the purchase price per unit is Rs. 1. Find the followings:

(i) Optimal lot size (EOQ).

(ii) Total cost.

(iii) No. of orders per year.

(iv) Time difference between consecutive two orders. $(2 + 2) + 4 = 8$

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