SP-III/Mathematics/301C-1C(T)/19

B.Sc. Semester III (General) Examination, 2018-19

MATHEMATICS

Course ID : 32118 **Course Code : SPMTH-301C-1C(T)**

Course Title : Algebra

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Find the modulus and amplitude of $\frac{3+5i}{2-3i}$.

(b) If *a, b, c* be positive real numbers, prove that
$(a^2b+b^2c+c^2a)(ab^2+bc^2+ca^2)\geq 9a^2b^2c^2$.

(c) If α be a multiple root of order 3 of the equation $x^4+bx^2+cx+d=0$ $(d\neq 0)$ show that $\alpha=-\frac{8d}{3c}$.

(d) A relation R is defined on the set $\mathbb{Z}$ by "aRb if and only if $ab>0$" for $a,b\in\mathbb{Z}$. Examine if R is an equivalence relation.

(e) Find the dimension of the subspace S of $\mathbb{R}^3$ where $S=\{(x,y,z): 2x+y-z=0\}$.

(f) Let $f,g:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=3x^2-5$ and $g(x)=\frac{x}{x^2+1}$. Then find *f*o*g* and *g*o*f*.

(g) Find *x* such that the rank of $A=\begin{pmatrix}2&1&4\\1&x&2\\4&0&x+2\end{pmatrix}$ is 2.

(h) Verify Cayley-Hamilton theorem for $A=\begin{pmatrix}1&4\\2&3\end{pmatrix}$.

2. Answer *any four* questions: 5×4=20

(a) If *a, b, c, d* be distinct positive real numbers and $S=a+b+c+d$ then prove that
$\frac{S}{S-a}+\frac{S}{S-b}+\frac{S}{S-c}+\frac{S}{S-d}>5\frac{1}{3}$.

(b) For what values of *K*, the following equations
$x+y+z=1, 2x+y+4z=K, 4x+y+10z=K^2$, have solutions and completely in each case.

(c) Solve the equation $2x^4-5x^3-15x^2+10x+8=0$, the roots being in geometric progression.

(d) (i) If *a* is prime to *b*, prove that $a + b$ is prime to *ab*.

(ii) Prove that the product of any three consecutive integers is divisible by 6. 2+3=5

(e) (i) Show that $\mathbb{N}$ and $\mathbb{Z}$ have the same cardinality.

(ii) Let *A*, *B* be both finite sets of *n* elements and a mapping $f: A \to B$ is injective. Prove that *f* is a bijection. 3+2=5

(f) Find the eigenvalues and the corresponding eigen vectors of the matrix

$$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}.$$

3. Answer *any one* question: 10×1=10

(a) (i) Solve : $x^3 - 12x + 65 = 0$

(ii) Use the theory of Congruences to show that $7|(2^{5n+3} + 5^{2n+3})$ for all positive integer *n*.

(iii) Apply Descartes' rule of signs to find the nature of the roots of the equation $2x^4 + 14x^2 + 7x - 8 = 0$ 4+3+3=10

(b) (i) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ and find A^{-1}. 2+3=5

(ii) State first principle of induction and using this principle prove that $2^n < n!$ for $n \in \mathbb{N}$ and $n \geq 4$. 2+3=5