B.SC. THIRD SEMESTER (PROGRAMME) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 32118**

Course Title: Algebra **Course Code: SP/MTH/301/C-1C**

Full Marks: 40 **Time: 2 Hour**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

1. **Answer *any five* of the following questions:** **(2x5=10)**

a) Using Principle of Mathematical Induction, prove that $3^{2n-1} + 2^{n+1}$ is divisible by 7 for all natural number n.

b) Prove that $\frac{(n+1)^n}{2^n} > n!$

c) Find the values of i^i

d) Apply Descartes' rule of sign to examine the nature of roots of the equation $x^6 + 4x^4 + 2x^2 + 4x + 1 = 0$.

e) State fundamental theorem of classical algebra.

f) Find the dimension of the subspace W of R^3 defined by

$W = \{(x, y, z) \in R^3 : x + y + z = 0\}$.

g) Determine the rank of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{bmatrix}$$

h) Use Cayley-Hamilton theorem to compute A^{-1} where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

2. **Answer *any four* of the following questions:** **(5x4=20)**

a) If $\alpha = cos\frac{2\pi}{n} + isin\frac{2\pi}{n}$ and if p is prime to n, prove that

$1 + \alpha^p + \alpha^{2p} + \cdots + \alpha^{(n-1)p} = 0$.

b) If α be a root of the equation $x^3 - 3x - 1 = 0$, prove that the other roots are $2 - \alpha^2$ and $\alpha^2 - \alpha - 2$.

c) How many different relations can be defined on a set with n elements? How many of these are reflexive? 3+2

d) Determine the conditions for which the system of equations has (a) only one solution, (b) no solution, (c) many solution

$$x + y + z = 1$$
$$x + 2y - z = b$$
$$5x + 7y + az = b^2.$$

e) Determine the linear mapping $T: R^3 \to R^2$ which maps the basis vectors (1,0,0), (0,1,0), (0,0,1) of R^3 to the vectors (1,1), (2,3), (3,2) of R^2 respectively. Hence find $T(6,1,1)$.

f) Find the eigen values and the corresponding eigen vectors of the real matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

3. Answer *any one* of the following questions: $\mathbf{(10 \times 1 = 10)}$

a) (i) Find the least positive residue in $2^{41} (mod\ 23)$

(ii) If n be a positive integer, prove that $\frac{1}{2\sqrt{(n+1)}} < \frac{1.3.5.....(2n-1)}{2.4.6.....2n} < \frac{1}{\sqrt{(2n+1)}}$

(iii) Solve by Cardan's method $x^3 - 27x - 54 = 0.$ 2+4+4

b) (i) In R^2, $\alpha = (3,1)$, $\beta = (2,-1)$. Determine $L\{\alpha, \beta\}$ and hence show that $L\{\alpha, \beta\} = R^2$.

(ii) If α is an eigen value of a real orthogonal matrix A, then prove that $\frac{1}{\alpha}$ is also an eigen value of A.

(iii) Obtain the fully reduced normal form of the matrix

$$\begin{pmatrix} 2 & 4 & 1 & 0 \\ 1 & 2 & 0 & 3 \\ 3 & 6 & 2 & 5 \end{pmatrix}$$

3+3+4=10

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