M.Sc.-III/Mathematics-303C/18

M.Sc. 3rd Semester Examination, 2018

MATHEMATICS

(Continuum Mechanics)

Paper : 303C

Course ID : 32153

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

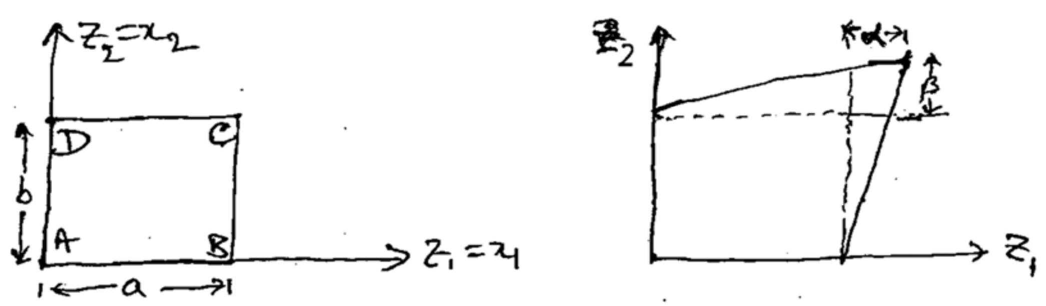
Notations and Symbols have their usual meanings.

A. Answer *any four* questions: 8×4=32

1. (a) Define deformation gradient and deformation tensor in Lagrangian coordinate system.

(b) Use these quantities to establish the conservation of mass in solid mechanics. 3+5=8

2. Consider a material undergoing an uniform deformation as shown in the figure below:



Determine:

(a) The displacement field

(b) The tangent vectors

(c) The strain tensor 3+3+2=8

3. (a) Prove that the strain tensor is symmetric, i.e. $\gamma_{ij} = \gamma_{ji}$.

(b) If γ_{ij} and γ'_{ij} are the strain tensors in old and new coordinate systems, then prove that $\gamma'_{ij} = a_{ir}\, a_{js}\, \gamma_{rs}$. 4+4=8

4. (a) Derive Cauchy's first law of motion associated with undeformed Cartesian coordinate system.

(b) Prove that the stress tensor is symmetric. 5+3=8

5. (a) Show that the principal stresses at a point are the roots of the equation $|\sigma_{ij} - \lambda\delta_{ij}| = 0$

(b) Consider the stress measurement data on a plane rotated from the plane of principal stresses given by,

$$\sigma_{ij} = \begin{bmatrix} 1 & \sqrt{3} & 0 \\ \sqrt{3} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Determine:

(i) Principal stress invariants,

(ii) Principal stresses. 4+4=8

6. (a) Prove that the generalized Hooke's law for small strain in linear elasticity is given by,

$$\sigma_{ij} = E_{ijkm}\,\gamma_{km}$$

(b) Prove that

$$E_{ijkm} = a_{ir}\,a_{js}\,a_{kt}a_{mu}\,E_{rstu}$$ 5+3=8

B. Answer *any one* question: 8×1=8

7. (a) State and prove the convection theorem for fluid motion.

(b) Use the above theorem to prove $div(\vec{u}) = 0$ for incompressible fluid flows. 4+4=8

8. (a) Derive the non-dimensional form of the continuity and the *x*-momentum equations representing the incompressible viscous fluid flows.

(b) Define different flow regimes based on Reynolds number. 5+3=8