POSTGRADUATE THIRD SEMESTER EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 32153**

Course Code: MATH 303C **Course Title: Continuum Mechanics**

Full Marks: 40 **Time:2 hours**

The figures in the margin indicate full marks

Symbols and notations have their usual meanings

GROUP - A

Answer *any three* of the following questions: **(8X3=24)**

1. Prove that the necessary and sufficient condition that a deformation of a body be a rigid body motion is the all components of strain tensor be zero throughout the body.
2. Prove that the extremum value of normal strains at a point of a continuum are the principal strain.
3. State generalized Hook's law. Hence find stress-strain relations for monoclinic elastic medium. 1+7
4. (a) Strain components e_{ij} are given as follows: $e_{11} = e_{22} = e_{33} = e_{12} = e_{13} = 0$, $e_{23} = x_2 x_3$, then find the reason such that there is no displacement field. 3

   (b) The strain tensor at a point in a solid is given by $[e_{ij}] = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 1 & -2 \\ -2 & -2 & 6 \end{bmatrix}$

   Determine the principle strains components and the corresponding principal direction of strain. 5
5. (a) If the state of stress at any point of a body be given by $\tau_{xx} = y^2 + \gamma(x^2 - y^2)$, $\tau_{zz} = (x^2 + y^2)$, $\tau_{yy} = x^2 + \gamma(y^2 - x^2)$, $\tau_{yz} = \tau_{zx} = 0$ and $\tau_{xy} = \tau_{yx} = f(x, y)$, determine the expression for τ_{xy} in order that the stress distribution is in equilibrium in the absence of body force. 3

   (b) Prove that the principal stresses are mutually orthogonal to each other. 5

GROUP – B

Answer *any two* of the following questions: **(8X2=16)**

6. (a) Derive the equation of continuity in Cartesian co-ordinates for homogeneous and in compressible fluid. 5

(b) Show that in a two dimensional incompressible steady flow field the equation of continuity is satisfied with the velocity components in rectangular co-ordinates given by $u(x,y) = \frac{k(x^2-y^2)}{(x^2+y^2)^2}$, $v(x,y) = \frac{2kxy}{(x^2+y^2)^2}$ where, k is an arbitrary constants. 3

7. For a viscous incompressible fluid, derive the Navier – Stokes equation and discuss the non-dimensionalization of Navier – Stokes equation. 5+3

8. (a) If the velocity of an incompressible fluid at a point (x,y,z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$ where, $r^2 = x^2+y^2+z^2$. Prove that motion is irrotational. Also find the velocity potential. 5

(b) Determine the restriction on f_1, f_2 and f_3 if $\frac{x^2}{a^2}f_1(t) + \frac{y^2}{b^2}f_2(t) + \frac{z^2}{c^2}f_3(t) = 1$ is a possible boundary surface of a liquid. 3

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