M.Sc.-III/Mathematics-302C/18

M.Sc. 3rd Semester Examination, 2018

MATHEMATICS

(Classical Mechanics)

Paper : 302C

Course ID : 32152

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Answer any five (05) questions: 8×5=40

1. (a) State and prove Poisson's theorem.

(b) Using the fundamental Poisson bracket find the values of α and β for which the system of equations $Q = q^{\alpha}\cos\beta p,\ P = q^{\alpha}\sin\beta p$ represents a canonical transformation. 5+3=8

2. (a) Find the path traversed by Foucault's pendulum. Deduce the expression for the time period of rotation of the plane of oscillation of the pendulum at latitude.

(b) Define the following terms with examples:

(i) Holonomic Constraint

(ii) Rheonomic Constraint (4+2)+(1+1)=8

3. (a) Find the Lagrange's equation of motion for a pendulum in Spherical Polar coordinates, of length *l*.

(b) State Hamilton's principle.

(c) Find the equation of motion of a simple pendulum using Hamilton's Procedure. 4+1+3=8

4. (a) Find the relation between the variations Δ and δ.

(b) If the transformation equations between two sets of coordinates are

$$P = 2\left(1+\sqrt{q}\cos p\right)\sqrt{q}\sin p$$

$$Q = \log(1+\sqrt{q}\cos p),$$

Show that

(i) the transformation is canonical.

(ii) the generating function of this transformation is

$$G = -(e^{Q}-1)^2\tan p$$

(c) What is restricted canonical transformation? 3+(1+3)+1=8

5. (a) Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$.

(b) Find the extremal of the functional $\int_0^1 \left(1 + Y'^2 + Z'^2\right)^{½} dx$ that satisfy the boundary conditions: $y(0) = 0$, $y(1) = 2$, $z(0) = 0$, $z(1) = 4$. 4+4=8

6. (a) Explain the effect of Coriolis force with an example.

(b) Derive Euler-Lagrange equation from the Hamilton's principle.

(c) Write down the Euler-Poisson equation and Euler-Ostrogradsky equation. 4+2+(1+1)=8

7. (a) Prove that $x^2 + y^2 + z^2 - c^2t^2$ is an invariant under Lorentz transformation.

(b) What is Lorentz-Fitzgerald contraction? Calculate the length of the rod of proper length 100cm when it is moving with velocity 0·6 cm/sec.

(c) Write down the basic postulates of relativity.

(d) Show that the relativistic kinetic energy of a particle of mass m is $T = (m - m_o)\, c^2$, where m_o is the rest mass of the particle. 2+(1+1)+2+2=8

8. (a) What is Brachistochrone problem? Show that the path followed by a particle in Brachistochrone problem is cycloid.

(b) Find the normal frequencies and normal coordinates of the system whose Lagrangian is given by

$$L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}\,(w_1^2\, x^2 + w_2^2\, y^2) + \alpha\, xy.$$ (1+3)+4=8

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