M.Sc.-III/Mathematics-301C/18

M.Sc. 3rd Semester Examination, 2018

MATHEMATICS

(Real Analysis)

Paper : 301C

Course ID : 32151

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Answer *any five* (05) questions: 8×5=40

1. (a) Let $f: [a, b] \to \mathbb{R}$ be a function of bounded variation. Then show that *f* is bounded on [*a*, *b*]. Is the converse true? Justify your answer.

(b) Consider the function $f(x) = x^2$, $x \in [-1, 1]$. Show that f is a function of bounded variation on [–1, 1].
Express *f* as the difference of two monotonically increasing functions on [–1, 1] with the help of its variation function. (2+2)+4=8

2. (a) Define the Riemann-Stieltjes integral $\int_a^b f(x).\, d\alpha(x)$.

(b) Prove that $f \in R(\alpha)$ on [*a, b*] if and only if for every $\in > 0$ there exists a partition P on [*a*, *b*] such that $\cup\,(P, f, \alpha) - L\,(P, f, \alpha) < \in$.

(c) Evaluate: $\int_0^3 xd[x]$. 2+4+2=8

3. (a) Let E be a countable subset of real numbers. Then prove that the outer measure of E is zero.

(b) Show that the outer measure is countably sub-additive.

(c) Prove that if m*(A) = 0, then m* (A∪B) = m*(B), where A,B are subsets of ℝ. 2+4+2=8

4. (a) Is the union of any countable collection of measurable sets measurable? Justify your answer.

(b) Construct a non-measurable subset of [0,1]. 4+4=8

5. (a) Define a measurable function.
Let *f* be a function defined on a measurable set E. Then prove that *f* is measurable if and only if for each open set 0, f^{-1} (0) is measurable.

(b) Let *g* be a measurable function on E and *f* be a continuous function on ℝ. Then prove that *fog* is a measurable function on E. Hence show that for a measurable function *f* with domain E, $|f|$ is also measurable there. (1+3)+(2+2)=8

6. (a) State and prove the Simple Approximation theorem for measurable functions.

(b) Give an example of a Lebesgue integrable function which is not Riemann integrable. (1+4)+3=8

7. (a) Let f and g be two bounded measurable function on a set E of finite measure. Then prove that $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$, where $\alpha, \ \beta \in \mathbb{R}$.

(b) State and prove Fatou's lemma for non-negative measurable functions. 4+(1+3)=8

8. (a) State Riemann-Lebesgue theorem.

(b) Define Lebesgue integral for a real-valued measurable function f on E.

(c) Find the value of the Lebesgue integration

$$\int_E f(x)dx \text{ where } f(x) = \begin{cases} 0, 0 \le x < 1 \\ 1, (1 \le x < 2) \cup (3 \le x < 4) \\ 2, (2 \le x < 3) \cup (4 \le x < 5) \end{cases}$$

(d) State the Lebesgue dominated convergence theorem. 2+2+2+2=8
