BCA 3rd Semester (Honours) Examination, 2021
BACHELOR OF COMPUTER APPLICATION

Course ID : 33314 **Course Code : BCA-GE-03**

Course Title : Mathematics-II

Time : 3 Hours **Full Marks : 80**

The figure in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group-A

1. **Answer all the questions from the following. Choose Correct Options:** **10 X 1 = 10**

i. $\lim_{x\to 0} \frac{|x|}{x}$ *is*

a) -1 b) 1 c) does not exist d) None of these

ii. The function $f(x) = \frac{x-|x|}{x}$ **is**

a) continues everywhere b) continues for all except Zero c) discontinues everywhere d) None of these

iii. The function $f(x) = x|x|$ **is**

a) discontinues b) continues but not differentiable at origin c) differentiable but not continuous at origin d) None of these

iv. **The conditions of Roll's theorem are**

a) sufficient b) necessary c) sufficient and necessary d) None of these

v. The value of $\int_0^1 \frac{dx}{2x^2+3x+1}$ **is**

a) $\log\frac{3}{8}$ b) $\log\frac{3}{2}$ c) $\log\frac{2}{3}$ d) None of these

vi. The Value of $\lim_{n\to\propto} \frac{1+2^{10}+3^{10}\ldots\ldots+n^{10}}{n^{11}}$ **is**

a) $\frac{1}{10}$ b) $\frac{1}{21}$ c) $\frac{1}{11}$ d) None of these

vii. The degree of the differential equation $x\frac{dy}{dx} + \left(\frac{d^2y}{dx^2}\right)^{2/3} = 0$ *is*

a) 1 b) 2 c) 3 d) None of these

viii. $y=x^2+x$ **is the solution of the differential equation**

a) (x +1)dx –dy=0 b) (2x +1)dx + dy =0 c) (2x+1)dx-dy=0 d) None of these

ix. The sequence $\left\{\frac{3+2\sqrt{n}}{\sqrt{n}}\right\}$

a) converges to 3 b) converges to 2 c) diverges d) None of these

x. The series $1 + r + r^2 + r^3 + \ldots\ldots\ldots\ldots$ **converges for**

a) r>1 b) r=1 c) r<1 d) None of these

Group-B

2. **Answer <u>any ten</u> questions from the following.** **10 X 2 = 20**

i. Show that $\lim_{(x,y)\to(0,0)} xy\frac{x^2-y^2}{x^2+y^2} = 0$

ii. Examine the equality of f_{xy} and f_{yx} , where $f_{(x,y)=x^3y+e^{xy^2}}$

iii. Examine the validity of the hypothesis and conditions of Roll's theorem for f(x) =x^3-4x on [-2,2]?

iv. Show that $\lim_{x\to 0}\frac{e^{1/x}-1}{e^{1/x}+1}$ **does not exist?**

v. Is f(x) continuous at x=1 where

F(x) = 2x, when $0 \le x < 1$

= 3 when x=1

= 4x when 1<x $\le$ 2 ?

vi. If y=log(1 + x), find nth derivative y_n .

vii. Examine the convergence of improper integral $\int_0^1 \frac{dx}{\sqrt{1-x}}$

viii. Evaluate :$\int \sqrt{\frac{x-1}{x+1}}\, dx$?

ix. Evaluate : $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$

x. Show that {x_n}, where x_n=1 $+\frac{1}{2}+\cdots+\frac{1}{n}$ cannot converge.

xi. Show that the infinite series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots \ldots converges.$

xii. Show that for any real number x $\lim_{n\to\propto}\frac{x^n}{n!}=0$

xiii. Solve the differential equation: $\frac{dy}{dx}=\frac{y-x}{y+x}$

xiv. If $f'(x)+f(x)=0\ and\ f(0)=2, find\ f(x)$

xv. Find the differential equation from the relation $x=a\ \cos t+b\sin t$ where a & b are arbitrary constant.

Group-C

3. **Answer <u>any four</u> questions from the following.** **5 X 4 = 20**

i. Discuss the derivability of the following function :

$f(x)=2x-3, 0\le x\le 2$

= $x^2-3, 2\le x\le 4$

at x=2,4

ii. State Lagrange's mean value theorem and verify for the function $f(x)=2x^2-7x+10$ on [2 , 5].

iii. Show that the series$\sum_{n=1}^{\propto}\frac{3.6.9\cdots 3n}{7.10.1\cdots(3n+4)}\ x^n, (x>0)$ converges for x$\le 1\ and\ diverges\ for\ x>1.$

iv. If $f(x,y) = \frac{x^2-xy}{x+y}$ $, when\ (x,y) \neq (0,0)$

= 0 when (x,y)=(0,0)

find $f_x\ (0,0)\ and\ f_y(0,0)$

v. Evaluate $\int_0^{\pi/4} \log(1 + tan\theta)\, d\theta$

vi. Solve the diff. equations: -

a) (x + y) (dx – dy) = dx + dy

b) y (1 +xy) dx + x (1 –xy) dy =0 5

Group-D

4. **Answer <u>any three</u> questions from the following.** **10 X 3 = 30**

a. i. Solve :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2; when\ x = 0, y = \frac{3}{8}\ and\ \frac{dy}{dx} = 1$$

ii. Evaluate :

$$\int_2^3 \frac{dx}{\sqrt{(x-1)(5-x)}}$$

b. i. Show that the series $1 - \frac{1}{3.2^2} + \frac{1}{5.3^2} - \frac{1}{7.4^2} \cdots\cdots is\ convergent.$

ii. Show that $\lim_{n\to\propto} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots\cdots + \frac{1}{\sqrt{n^2+n}}\right] = 1$

c. i. Examine for convergence the improper integral $\int_0^{\alpha} x^3 e^{-x^2} dx$

ii. Solve:

$$(6x - 5y + 4)dy + (y - 2x - 1)\, dx = 0$$

d. i. Establish the inequality:

$$x - \frac{x^2}{2} + \frac{x^3}{3(1+x)} < \log(1 + x) < x - \frac{x^2}{2} + \frac{x^3}{3}\ , x > 0$$

ii. Evaluate : $\operatorname{lt}_{x\to 0} \frac{\sqrt{4+x}-2}{x}\ and\ \lim_{x\to 1} \frac{x^2-1}{x-1}$

e. i. Assuming the validity of expansion, show that

$$e^x \cos x = 1 + x - \frac{2x^3}{3!} - \frac{2^2x^4}{4!} - \frac{2^3x^5}{5!} + \cdots$$

ii. State and prove Euler's theorem for homogeneous function of two variables.

f. i. Show that the equation of the curve whose slope at any point is equal to $y + 2x$ and which passes through the origin is $y=2(e^x - x+1)$

ii. Evaluate :

$$\int_0^{\pi} \frac{x\ \sin x}{1+ \cos^2 x} dx$$

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