SP-II/Mathematics/201/C-1B/19

B.Sc. 2nd Semester (Programme) Examination, 2019

MATHEMATICS

(Real Analysis)

Paper : 201/C-1B

Course ID : 22118

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Define limit point of a set. Give an example.

(b) Show that $N \times N$ is countable, where N is the set of natural numbers.

(c) Show that $S = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots\right\}$ is not a closed set.

(d) Show that $\lim_{n\to\infty} n^{1/n} = 1$.

(e) Show that the sequence $\left\{\frac{n+3}{2n+1}\right\}$ is bounded.

(f) Give an example of a bounded sequence which is not convergent.

(g) Show that the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{n}{n+1} + \cdots$ does not converge.

(h) Examine the following sets are compact or not:

(i) (1, 2] (ii) [2, 3].

2. Answer *any four* questions: 5×4=20

(a) (i) Show that a point α is a limit point of a set S if and only if every neighbourhood of α contains infinitely many points of S.

(ii) Show that every point of a finite set is an isolated point. 3+2=5

(b) (i) State Bolzano Weistrass theorem and verify it for the set $S = \left\{\frac{n}{n+1} : n \in N\right\}$.

(ii) Show that union of two closed sets is closed. 3+2=5

(c) (i) Show that a convergent sequence is bounded.

(ii) State Cauchy's general principle of convergence of a real sequence. 3+2=5

(d) (i) Define compact set. Give example.

(ii) Show that every finite subset of $\mathbb{R}$ is compact. 2+3=5

(e) If $\lim_{n\to\infty} x_n = l$ and $\lim_{n\to\infty} y_n = m$, then show that $\lim_{n\to\infty} (x_n - y_n) = l - m$.

(f) State Cauchy's root test for a series of positive terms and also test the convergence of the series $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{4^3} + \cdots + \frac{1}{(n+1)^n} + \cdots\cdots$

3. Answer *any one* question: 10×1=10

(a) (i) Show that the set of rational numbers is countable.

(ii) Find the upper and lower limits of the sequence $\left\{(-1)^n + \sin\frac{n\pi}{4}\right\}_n$.

(iii) Show that $1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \cdots$ is convergent. 4+3+3=10

(b) (i) Show that the sequence $\left\{\frac{1}{n}\right\}$ is a Cauchy sequence.

(ii) Test the convergence of the series $\frac{1}{1\cdot 2^2} + \frac{1}{2\cdot 3^2} + \frac{1}{3\cdot 4^2} + \cdots$.

(iii) If S be a bounded set and T be a set such that $T = \{-x \mid x \in S\}$. Show that T is also bounded and $\sup T = -\inf S$ and $\inf T = -\sup S$. 2+3+5=10