M.SC. SECOND SEMESTER EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 22165**

Course Code: Math-205C(IA)

Course Title: Integral Transforms & Computational Methods for PDEs

Full Marks: 32 **Time: 2 Hours**

The figures in the margin indicate full marks

Symbols and notations have their usual meaning

Group - A

Answer *any two* of the following questions: $\mathbf{8 \times 2 = 16}$

1. Using Fourier transform, solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 < x < \pi, 0 < y < y_0$ under the boundary conditions $u(0, y) = 0, u(\pi, y) = 1, u_y(x, 0) = u(x, y_0) = 0.$ 8

2. a) Find $L^{-1}\left\{\frac{w}{s^2+w^2}\right\},\ s > 0$ using complex inversion formula.

   b) Use Laplace transform technique to solve the ODE:

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = \sin t,\ \ t \geq 0$$

   subject to the conditions $x(0) = x'(0) = 0.$ 4+4=8

3. (a) Derive transformation and inverse transformation formula for Hankel transformation.

   (b) Find the Hankel transformation of $f(x) = \begin{Bmatrix} a^2 - x^2\ , 0 < x < a, n = 0 \\ 0,\ \ x > a, n = 0 \end{Bmatrix}$ (2+2)+4=8

Group - B

Answer *any two* of the following questions: $\mathbf{8 \times 2 = 16}$

4. (a) Define the following terms: (i) Well-posed problems and classical solutions of PDE, (ii) Computational grid, (iii) Stencil of a scheme.

   (b) Construct the FTCS-scheme (forward in time and central in space) for the one-dimensional transport equation: $u_t + a\, u_x = bu_{xx}$ where a and b are constants.

(2+1+1)+4=8

5. Use the Crank-Nicolson Method to calculate a numerical solution of the equation $u_t = u_{xx}$, $0 < x < 1,\ t > 0$, where (1) $u = 0,\ x = 0$ and $1, t \geq 0$, (ii) $u = 2x$, $0 \leq x \leq \frac{1}{2}$, $t = 0$, (iii) $u = 2(1 - x),\ \frac{1}{2} \leq x \leq 1$, $t = 0$ $\left(\text{Take } \Delta x = \frac{1}{10},\ \Delta y = \frac{1}{100}\right)$. 8

6. A tightly stretched string with fixed end points $x = 0$ and $x = 1.0$ is at rest in its equilibrium position. At $t = 0$ each point of the string is given a velocity $20x(1 - x)$. Find the displacement of the string at $x = 0.6$ and $t = 0.2$ by finite difference method, taking $\Delta x = 0.2$ and $\Delta t = 0.1$. Consider the normal form

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

for vibration of string. 8

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