M.SC. SECOND SEMESTER EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 22154**

Course Code: Math-204C

Course Title: Techniques of Applied Mathematics (Generalized Functions, Special Functions, Integral Equations)

Full Marks: 40 **Time: 2 Hours**

<u>The figures in the margin indicate full marks</u>

Symbols and notations are their usual meaning

Answer *any FIVE of* the following questions: **8x5=40**

1. What do you mean by support of generalized function? Also prove that $\lim_{\varepsilon\to 0+}\frac{1}{x-i\varepsilon} = \pi i\delta(x) + \mathrm{P}.\frac{1}{x}$. 3+5

2. What is a regular generalized function? Show that $\mathrm{P}.\frac{1}{x}\ \epsilon\mathcal{D}(\mathcal{R})$ defined by $(\mathrm{P}.\frac{1}{x},\varphi) = \lim_{\varepsilon\to\infty}[\int_{-\infty}^{\infty}\frac{\varphi(x)}{x}dx + \int_{\varepsilon}^{\infty}\frac{\varphi(x)}{x}dx]$, $\varphi\epsilon\mathcal{D}(\mathcal{R})$ is a generalized function. 3+5

3. **i)** Prove that $\int_0^p J_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$ *and* for any number $p > 0$.

ii) Prove that $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$ 5+3

4. i) Define Legendre equation and Legendre's polynomial of order n. 1+3

ii) If $f(x) = \begin{Bmatrix} 0\,, -1 < x \leq 0 \\ x,\ \ 0 < x < 1 \end{Bmatrix}$,

then prove that $f(x)=\frac{1}{4}P_0(x) + \frac{1}{2}P_1(x) - \frac{5}{16}P_2(x) - \frac{3}{32}P_1(x) + \cdots$ 4

5. Define Bessel's function of second kind and prove that the Bessel function of the first kind of order n is the coefficient of z^n in the expansion of $e^{\frac{x}{2}(z-\frac{1}{z})}$. 2+6

6. If the function $K(x,t)$ be continuous on the rectangle $R = \{(x,t); a \leq x, t \leq b\}$, $K(x,t)$ possesses continuous partial derivative $K_x(x,t)$ in R and $\varphi(x) = \int_a^x K(x,t)dt$, $a \leq x \leq b$ then show that for all x in $[a,b]$, $\varphi'(x) = \int_a^x K_x(x,t)dt + K(x,x)$. Also solve the integral equation $u(x) = cos\ x + (x-2) + \int_0^x (t-x)u(t)dt$. 5+3

7. Using the Laplace transform method, solve the following integral equation –

(i) $u(t) = a\, sin\, t + 2\int_0^t cos\,(t-s)u(s)ds$

(ii) $u(t) = a\, cos\, t + 2\int_0^t \sin\,(t-s)\, u'(s)ds$ 4+4

8. (i) Differentiate between Fredholm and Volterra integral equations.

(ii) Solve the following integral equation by the method of successive approximation

$y(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xty(t)dt.$ 4+4

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