M.SC. SECOND SEMESTER EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 22152**

Course Code: Math-202C **Course Title: Topology**

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

Answer *any Five* of the following questions: **(8×5)**

1. (a) Let (X, τ) be a topological space. Prove that a collection $\mathcal{B}$ of open sets of X constitute a base of X if and only if (i) $\phi \in \mathcal{B}$, (ii) X is the union of members of $\mathcal{B}$ (iii) whenever $U, V \in \mathcal{B}$ and $x \in U \cap V$, there exists a member $B \in \mathcal{B}$ such that $x \in B \subset U \cap V$. 4

   (b) Construct two distinct topologies τ_1 and τ_2 other than discrete and indiscrete topologies on $X = \{a, b, c\}$ such that $\tau_1 \subset \tau_2$. 2

   (c) Let Y be a subspace of the topological space X and $A \subset Y$. Prove that $Cl_Y(A) \subset Cl_X(A)$. 2

2. (a) Prove that a topological space *X* is normal if and only if for each closed set F and each open set *G* in *X* with $F \subset G$ there exists an open set *H* such that $F \subset H \subset \overline{H} \subset G$. 4

   (b) Let $(\mathbb{R}, \tau)$ be the usual topological space. Find out two open sets U, V in $\mathbb{R}$ such that $1 \in U, [2,5] \subset V$ and $U \cap V = \emptyset$, if exist. 2

   (c) Prove that every singleton in a T_1 topological space is closed. 2

3. (a) Let S be a set of topological spaces. Define an equivalence relation with justification on S. 4

   (b) Give an example with justification to show that $(A \cup B)^{\circ} = A^{\circ} \cup B^{\circ}$ need not be true. 2

   (c) Give an example with justification of a topological space X and a subspace of A such that an open set in A is not open in X. 2

4. (a) Show that a subset *A* of a topological space *X* is open if and only if $Bd(A) \subset X - A$ where *Bd*(A) denotes the boundary of *A*. 3

   (b) Suppose X is a topological space and A is a subspace of X. Construct with justification a basis of the subspace A in terms of a basis of X. 3

(c) Find the exterior and boundary of the set $[2,5) \cup (7,8)$ with respect to the usual topology on $\mathbb{R}$. 2

5. (a) Suppose X and Y are two topological spaces, and $f: X \to Y$ is a function. Prove that f is continuous if and only if $f\left(Cl_X(A)\right) \subset Cl_Y(f(A))$ for each $A \subset X$. 3

(b) If A is compact and B is closed in a regular topological space X, then prove that there exist open sets U, V in X such that $A \subset U, B \subset V$ and $U \cap V = \emptyset$. 3

(c) Give an example with justification to show that a regular space need not be a normal space. 2

6. (a) State and prove Pasting Lemma. 4

(b) For any two subsets A and B of a topological space X, prove that $Cl(A) \cap Int(B) = Cl(A \cap B) \cap Int(B)$. 3

(c) Let $X = \{0,1\}$ and $\tau = \{\emptyset, X, \{1\}\}$. Find out a dense subset of X, if exists. 1

7. (a) If *E* is a connected subspace of a topological space *X* and *F* is a subset of *X* such that $E \subset F \subset \bar{E}$, prove that *F* is also a connected subspace of *X*. 4

(b) Let X and Y be two topological spaces, and $f: X \to Y$ be a one-one and onto mapping. Prove that f is homeomorphism if and only if $f(Int_X(A) = Int_Y(f(A))$ where A is a subset of X. 4

8. (a) What is meant by finite intersection property? Prove that each collection of closed sets in a compact space X with finite intersection property has a nonempty intersection. 1+3

(b) Write down one limitation for each of open and continuous functions. 2

(c) Prove or disprove: the usual topological space in $\mathbb{R}$ is connected. 2

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