SH-II/Mathematics/203/GE-2/19

B.Sc. 2nd Semester (Honours) Examination, 2019

MATHEMATICS

(Real Analysis)

Paper : 203/GE-2

Course ID : 22114

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) If $S = \left\{\frac{5}{n} : n \in N\right\}$, using Archimedean property show that $Inf\ S = 0$.

(b) Show that the set of all natural numbers is not bounded above.

(c) Using Cauchy's general principle of convergence, show that the sequence $\left\{\frac{n}{n+1}\right\}_n$ is convergent.

(d) Show that the sequence $\{2^n : n \in N\}$ is not a Cauchy sequence.

(e) Find the derived set of the set $\left\{1 + \frac{1}{2n} : n \in \mathbb{N}\right\}$.

(f) Define open set. Give example.

(g) Show that the series $\sum \frac{2\sqrt{n}}{n^2+1}$ converges.

(h) If a series $\sum u_n$ converges, then show that $\lim\limits_{n\to\infty} u_n = 0$.

2. Answer *any four* questions: 5×4=20

(a) If $\lim\limits_{n\to\infty} x_n = l$ and $\lim\limits_{n\to\infty} y_n = m$, then show that $\lim\limits_{n\to\infty} (x_n y_n) = lm$.

(b) (i) Show that the derived set of any set in $\mathbb{R}$ (the set of real numbers) is closed.

(ii) Find the derived set of the set S, where $S = \left\{\frac{1}{3^m} + \frac{1}{3^n} : m, n \in N\right\}$. 3+2=5

(c) (i) Let $\{u_n\}_n$ be a null sequence and $\{v_n\}_n$ be bounded. Then show that $\{u_n v_n\}_n$ is also a null sequence.

(ii) Apply Sandwith theorem to show that the sequence $\{u_n\}_n$, where $u_n = \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}$ converges to 1. 3+2=5

(d) (i) If $\sum u_n$ be a convergent series of positive terms, prove that $\sum u_n^2$ is also a convergent series.

(ii) Test the convergence of $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$. 3+2=5

(e) (i) Define alternating series and state Leibnitz's test for convergence of an alternating series.

(ii) Show that the series $\sum_{n=1}^{\infty}(-1)^{n-1}n^{-1/2}$ is convergent. Examine its absolute convergence. 2+3=5

(f) Define Cauchy sequence. Show that every Cauchy sequence is convergent. 1+4=5

3. Answer *any one* question: 10×1=10

(a) (i) Show that the set of all odd integers is enumerable.

(ii) Show that every compact subset of $\mathbb{R}$ is closed.

(iii) Show that the following series converges conditionally:

$\frac{3}{1\cdot 2} - \frac{5}{2\cdot 3} + \frac{7}{3\cdot 4} - \cdots$. 3+4+3=10

(b) (i) If S and T be any two sets, then show that $\overline{S \cap T} \subset \overline{S} \cap \overline{T}$ but the converse is not true.

(ii) Let $\{u_n\}$ be a convergent sequence of real numbers converging to u. Then show that the sequence $\{|u_n|\}$ converges to $|u|$.

(iii) Test the convergence of the series $\frac{1}{1\cdot 2^2} + \frac{1}{2\cdot 3^2} + \frac{1}{3\cdot 4^2} + \cdots$. 3+4+3=10

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