B.Sc. 2nd Semester (Honours) Examination, 2019

MATHEMATICS

(Differential Equations and Vector Calculus)

Paper : 202/C-4

Course ID : 22112

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Find a unit vector, in the plane of the vectors $\hat{\imath}+2\hat{\jmath}-\hat{k}$ and $\hat{\imath}+\hat{\jmath}-2\hat{k}$, which is perpendicular to the vector $2\hat{\imath}-\hat{\jmath}+\hat{k}$.

(b) The differential equation $(4x+3y+1)dx+(3x+2y+1)dy=0$ with $y(0)=1$ represents a family of (i) Circles, (ii) Parabolas, (iii) Ellipses, (iv) Hyperbolas. Choose the correct answer with proper justification.

(c) Determine if the following functions are linearly dependent or linearly independent:

$f(x)=4^x, g(x)=4^{x+2}$, using Wronskian

(d) Find out the singular point(s) of the differential equation $(x-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\frac{1}{x}y=0$.

(e) If $\vec{r}=3t\hat{\imath}+3t^2\hat{\jmath}+2t^3\hat{k}$, then find $\frac{d\vec{r}}{dt}\times\frac{d^2\vec{r}}{dt^2}$.

(f) Find the equilibrium points of the system

$\frac{dx}{dt}=y^2-5x+6$

$\frac{dy}{dt}=x-y$.

(g) If $\vec{\alpha}\times\vec{\beta}=\vec{\beta}\times\vec{\gamma}=\vec{\gamma}\times\vec{\alpha}$, then show that the vectors $\vec{\alpha},\vec{\beta}$ and $\vec{\gamma}$ are coplanar.

(h) Find the solution of the equation $l\frac{d^2\theta}{dt^2}+g\theta=0$, satisfying $\theta=\alpha$ and $\frac{d\theta}{dt}=0$ when $t=0$.

2. Answer *any four* questions: 5×4=20

(a) Show that the function $f(x,y)=y^{\frac{1}{2}}$ does not satisfy the Lipschitz condition on $S:|x|\le 1, 0\le y\le 1$, but satisfies Lipschitz condition on $S:|x|\le a, b\le y\le c\ (a,b,c>0)$.

(b) Solve: $(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(1+2x)^2$.

(c) Solve: $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$ by method of variation of parameter.

(d) If $\vec{a} = 2t^2\hat{\imath} + 3(t-1)\hat{\jmath} + 4t^2\hat{k}$ and $\vec{b} = (t-1)\hat{\imath} + t^2\hat{\jmath} + (t-2)\hat{k}$, then find $\int_0^2 (\vec{a}\,.\vec{b})dt$ and $\int_0^2 (\vec{a} \times \vec{b})dt$.

(e) Using the method of undetermined coefficients solve the equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = x + e^x \sin x$.

(f) Determine the nature of the equilibrium point of the linear system:

$$\left.\begin{aligned}\frac{dx}{dt} &= x\\ \frac{dy}{dt} &= y\end{aligned}\right\}.$$

Also sketch the corresponding phase portrait in the phase plane. 2+3=5

3. Answer *any one* question: 10×1=10

(a) (i) Obtain the power series solution of the initial value problem $(x^2-1)y'' + 3xy' + xy = 0$, given $y(0) = 4,\ y'(0) = 6$, where $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$.

(ii) If $\vec{x} = 3a^2\hat{\imath} - (a+4)\hat{\jmath} + (a^2 - 2a)\hat{k}$ and $\vec{y} = \sin a\,\hat{\imath} + 3e^{-a}\hat{\jmath} - 3\cos a\,\hat{k}$, then prove that $\frac{d^2}{da^2}(\vec{x} \times \vec{y}) = -30\hat{\imath} + 14\hat{\jmath} + 20\hat{k}$ at $a = 0$. 6+4=10

(b) (i) Solve the following linear system:

$$\frac{dx}{dt} = 6x - 3y$$

$$\frac{dy}{dt} = 2x + y\,.$$

(ii) Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{3x}$

(iii) For the curve $\vec{r} = (2a\,cos\,t, 2a\,sin\,t, bt^2)$, show that $[\dot{\vec{r}}\ \ddot{\vec{r}}\ \dddot{\vec{r}}] = 8a^2bt$. 4+3+3=10

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