B.Sc. Semester I (General) Examination, 2018-19
MATHEMATICS

Course Id : 12118   **Course Code : SPMTH-101C-1A(T)**

Course Title : Calculus, Geometry & Differential Equation

Time: 2 Hours   **Full Marks: 40**

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

1. Answer *any five* questions: 2×5=10

   (a) Examine the curve $y - \sin x$ regarding its convexity or concavity to the x-axis.

   (b) Evaluate: $\lim\limits_{x \to 0} \dfrac{\sin x - x}{x^3}$

   (c) Find the nature of the conic represented by $9x^2 - 6xy + y^2 - 14x - 2y + 12 = 0$

   (d) Identify the order and degree of the differential equation $\sqrt{1 + (y')^2} = x + 1$

   (e) Evaluate: $\int_0^{\frac{\pi}{2}} \cos^4 x\, dx$

   (f) Find the asymptotes of $x^2 - y^2 = 9$

   (g) Find an integrating factor of the differential equation $(x^2 + y^2 + 2x)dx + 2y\,dy = 0$.

   (h) Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$.

2. Answer *any four* questions: 5×4=20

   (a) Evaluate: $\lim\limits_{x \to 0} (\cos x)^{\cot^2 x}$ 5

   (b) If $y = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}, |x| < 1$, then show that

      (i) $(1 - x^2)y_2 - 3xy_1 - y = 0$

      (ii) $(1 - x^2)y_{n+2} - (2n + 3)xy_{n+1} - (n + 1)^2 y_n = 0$ 2+3=5

   (c) Solve: $(4x^2 y - 6)dx + x^3 dy = 0$ 5

   (d) Find the surface area generated by revolving the straight line $x = 1 - y, 0 \le y \le 1$ about y axis.   5

   (e) Find the asymptotes of $y^2 - x^2 - 2x - 2y - 3 = 0$.   5

   (f) What is rotation of axes? What will be the form of the equation $x^2 - y^2 = 4$, if the co-ordinate axes are rotated through an angle $\left(-\dfrac{\pi}{2}\right)$.   1+4=5

3. Answer *any one* question: 10×1=10

(a) (i) Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.

(ii) Find if there is any point of inflexion on the curve $y - 3 = 6(x - 2)^5$

(iii) Solve: $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$ 4+3+3=10

(b) (i) Find a and b such that $\lim\limits_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$.

(ii) Find the equation of the right circular cylinder whose radius is 1 and x-axis is the axis. 5+5=10

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