B.SC. FIRST SEMESTER (PROGRAMME) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 12118**

Course Code: SP/MTH/101/C-1A

Course Title: Calculus, Geometry & Differential Equations

Full Marks: 40 **Time: 2 Hours**

<u>The figures in the margin indicate full marks</u>

Notations and symbols have their usual meaning

1. **Answer *any five* of the following questions:** $5 \times 2 = 10$

a) Find the envelop of the family of circles $x^2 + y^2 - 2ax\cos\alpha - 2ay\sin\alpha = c^2$, where α is the parameter, and interpret the result.

b) Find the range of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Also determine the point of inflection.

c) Find the equation of a right circular cylinder of radius 2 whose axis passes through $(1,2,3)$ and has direction cosines proportional to $(2,-3,6)$.

d) Find the nature of the conic $\frac{8}{r} = 4 - 5\cos\theta$.

e) Solve $x\cos\frac{y}{x}(ydx + xdy) = y\sin\frac{y}{x}(xdy - ydx)$.

f) If $\lim_{x\to a}\left\{\frac{f(x)}{g(x)}\right\}$ exists finitely and $\lim_{x\to a} g(x) = 0$, then prove that $\lim_{x\to a} f(x) = 0$.

g) State Leibnitz's Theorem on successive derivatives.

h) Evaluate: $\int_0^{\frac{\pi}{2}} \sin^5 x\, dx$ using reduction formula.

2. **Answer *any four* of the following questions:** $5 \times 4 = 20$

a) Solve: $xy - \frac{dy}{dx} = y^3 e^{-x^2}$

b) If $x = Ae^{-\frac{kt}{2}}\cos(pt + \epsilon)$, then prove that $\frac{d^2x}{dt^2} + k\frac{dx}{dt} + n^2x = 0$, where $n^2 = p^2 + \frac{1}{4}k^2$.

c) Deduce the necessary and sufficient condition of tangency of a plane to a sphere.

d) Find the asymptotes of $x^3 + 4x^2y + 4xy^2 + 5x^2 + 5xy + 10y^2 - 2y + 1 = 0$.

e) Prove that the two conics $\frac{l_1}{r} = 1 - e_1\cos\theta$ and $\frac{l_2}{r} = 1 - e_2\cos(\theta - \alpha)$ will touch one another if $l_1^2(1 - e_2^2) + l_2^2(1 - e_1^2) = 2l_1l_2(1 - e_1e_2\cos\alpha)$.

f) i) Determine $\lim_{x\to a}\left(2 - \frac{x}{a}\right)^{\tan\frac{\pi x}{2a}}$

ii) If $y = \cos(m \sin^{-1} x)$ show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$ and hence find $y_n(0)$. 2+3

3. Answer *any one* of the following questions: $\mathbf{10 \times 1 = 10}$

a) Is integrating factor of a differential equation unique? Justify your answer. Show that e^{x^2} is an integrating factor of $(x^2 + xy^4)dx + 2y^3dy = 0$ and hence solve it. 1+4+1+4

b) i) Find $\lim_{x\to 0} \frac{1-\sin x-\cos x+l \quad (1-x)}{x tan^2(x)}$.

ii) Find the point of inflection of the curve $y = (logx)^3$.

iii) To what point the origin is to be moved so that one can get rid of the first degree terms from the equation $x^2 + xy + 2y^2 - 7x - 5y + 12 = 0$. 3+3+4

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