M.Sc. 1st Semester Examination, 2018

PHYSICS

Course Title : Mathematical Methods-I & Classical Mechanics

Paper : PHYS 101C

Course ID : 12451

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unit – I

1. Answer *any three* of the following: 2×3=6

(a) Write down the values of i^i in the form $a + ib$.

(b) Show that, the matrix $B^{\dagger}AB$ is Hermitian or skew–Hermitian according as A is Hermitian or skew–Hermitian. 2

(c) Identify (with propor mathematical justification) the type of singularity of the function $z^2 \exp\left(\frac{1}{z}\right)$ at $z = 0$. 2

(d) Show that, the complex variable function $f(z) = |z|^2$ is differentiable only at the origin. 2

(e) Evaluate, $\int_c \frac{dz}{(z^2-1)}$, where c is the circle $x^2 + y^2 = 4$. 2

2. Answer *any two* of the following: 4×2=8

(a) Prove that, the trace and the determinant of a finite-dimensional matrix can be written respectively as, the sum and the product of its eigenvalues. 4

(b) Obtain the Laurent series expansion of the function $\int(z) = \frac{z}{z^2-3z+2}$ valid for each of the following domain defined by

(i) $D_1 = \{Z \in C; |Z| < 1\}$

(ii) $D_2 = \{Z \in C; 1 < |Z| < 2\}$ 2+2=4

(c) Let $y_1 = 5x_1 + 3x_2 + 3x_3$, $y_2 = 3x_1 + 2x_2 - 2x_3$, $y_3 = 2x_1 - x_2 + 2x_3$ be a linear transformation from (x_1, x_2, x_3) to (y_1, y_2, y_3) and $z_1 = 4x_1 + 2x_3$, $z_2 = x_2 + 4x_3$, $z_3 = 5x_3$ be a linear transformation from (x_1, x_2, x_3) to (z_1, z_2, z_3). Find the linear transformation from (z_1, z_2, z_3) to (y_1, y_2, y_3) by inverting appropriate matrix and matrix multiplication. 4

(d) Define Holomorphic function. Find all $v(x, y): R^2 \to R^2$, such that for $z = x + iy$ the function $f(z) = (x^3 - 3xy^2) + iv(x, y)$ is holomorphic. 1+3=4

3. Answer *any one* of the following: 6×1=6

(a) Using Cauchy's Residue theorem evaluate any one of the following integral:

(i) $\int_0^\infty \frac{\log(1+x^2)}{(1+x^2)}\, dx$

(ii) $\int_0^{2\pi} \frac{d\theta}{a+bcos\theta}$ 6

(b) (i) Is the system of vectors $X_1 = (2, 2, 1)^T, X_2 = (1, 3, 1)^T, X_3 = (1, 2, 2)^T$ linearly dependent?

(ii) Let $f(z) = \cos x - isinhy$, where $z = x + iy$ is a complex variable defined by the whole complex plane. For what value (s) of z does $F'(z)$ exist? Let, $F(z)$ be an analytic function. If $F'(z)$ is identically zero in a given domain Ω, then show that $F(z)$ is constant in Ω. 2+4=6

Unit – II

1. Answer *any three* of the following: 2×3=6

(a) Show that energy is conserved in cases where the Hamiltonian is time-independent. 2

(b) Define Poisson bracket. 2

(c) State and explain Liouville's Theorem. 2

(d) What are Action-angle coordinates? Write its application. 2

(e) A particle moves with velocity *v* in an elliptical path in an inverse square force field *U(r)* = –*k*/*r*. Show that v^2 = (*k*/*m*) (2/*r* – 1/*a*). 2

2. Answer *any two* of the following: 4×2=8

(a) From Lagrange's equation of motion deduce Hamilton's equations of motion, using the definitions for the generalized momenta and Hamiltonian. 4

(b) Derive the expression for *integral invariants of Poincaé.* 4

(c) Explain strain and stress tensor. Write down their property. 4

(d) Consider a pendulum of mass *m* and length *l* whose point of support is rapidly oscillating in the vertical direction with *y* = –*a* cos *ωt*. Obtain the Lagrangian and the equation of motion of the system. 4

3. Answer *any one* of the following: 6×1=6

(a) Proof the Jacobi's identity. Show that Poisson bracket is invariant under canonical transformations. 3+3=6

(b) Consider a particle in three dimension (x, y, z), subject to a central force based potential

$$V(x, y, z) = V\left(\sqrt{(x^2 + y^2 + z^2)^2}\right)$$

Show that all three components of angular momentum are conserved. The Poisson bracket displays its true power in the search for constants of the motion—Explain. 4+2=6