M.Sc.-I/Math-105-IA/18

M.Sc. 1st Semester Examination, 2018

MATHEMATICS

Paper : 105-IA (Numerical Analysis)

Course ID : 12195

Time: 1 Hour **Full Marks: 16**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Answer *any two* questions. 8×2=16

1. (a) Consider the linear system of equations $Ax = b$ where A is a matrix of dimension $n \times n$ and x and b are vectors of dimension $n \times 1$.

Formulate the Gauss-Seidel iteration scheme $x^{(K+1)} = Bx^{(K)} + C$, $K = 0, 1, 2, \ldots$ with the iteration matrix B which depends on A and the new right hand side (a column vector) C. Write down the explicit form of B in terms of L, U and D where

$$A = L + D + U.$$

(b) (i) What will be the form of B for Jacobi iteration scheme?

(ii) Consider the system of equations

$$2x - 2y = 7$$
$$-x + 2y - z = 1$$
$$-y + 2z = 1.$$

Use Gauss-Seidel scheme to perform 2 iterations for this system starting with

$$x^{(0)} = (0, 0, 0)^T.$$ 4+(1+3)=8

2. (a) Derive the Newton's iteration scheme to solve the non-linear algebraic equation

$$f(x) = 0.$$

(b) Apply Newton's iteration scheme to find the (approximate) root of the equation

$$f(x) = \cos x - xe^x = 0$$

starting with $x_0 = 1$.

List the values of x_K, $\Delta x_K, x_{K+1}$ and $f(x_{K+1})$ for $K = 0, 1$ and 2. 4+4=8

3. Use the Euler method to solve numerically the initial value problem (IVP)

$\frac{du}{dt} = -2tu^2$, $u(0) = 1$, $t \in [0,1]$ with $h = 0{\cdot}2$.

Determine the percentage of relative error at $t = 1$ when exact solution at $t = 1$ is 0·5. 6+2=8

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