M.Sc.-I/Math-104C/18

M.Sc. 1st Semester Examination, 2018

MATHEMATICS

Paper : 104C (Techniques of Applied Mathematics)

Course ID : 12154

Time: 2 Hours **Full Marks: 40**

The questions are of equal value.
The figures in the right hand side margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meanings.

Answer *any five* questions. 8×5=40

1. (a) Verify that the origin is a regular singular point of $2x^2y'' + xy' - (x+1)y = 0$ and find two independent Frobenius series solutions of it.

(b) Express $2 - 3x + 4x^2$ in terms of Legendre Polynomial. 6+2=8

2. (a) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2}{2n+1} & \text{if } m = n \end{cases}$

(b) If $\Phi(x)$ vanishes only when $x = x_1, x_2, \dots, x_n$, then show that $\delta\left(\Phi(x)\right) = \sum_{i=1}^{n} \frac{\delta\left(x - x_i\right)}{\left|\Phi'\left(x_i\right)\right|}$.

5+3=8

3. (a) Show that $x\, \mathrm{J}_n^1(x) = n\mathrm{J}_n(x) - x\, \mathrm{J}_{n+1}(x)$.

(b) Evaluate : $\int_{-\infty}^{\infty} \frac{1+e^{2t}\delta(t+1)}{1+t^2} dt$ 5+3=8

4. (a) Reduce the following boundary value problem into an integral equation

$\frac{d^2y}{dx^2} + \lambda y = 0$ with $y(0) = 0; y(l) = 0$.

(b) Solve the Fredholm integral equation of the second kind $y(x) = x + \lambda \int_0^1 (xt^2 + x^2t)y(t)dt$.

4+4=8

5. (a) Solve the following integral equation by the method of successive approximations

$y(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xt\, y(t)dt$.

(b) Determine $D(\lambda)$ of the integral equation $y(x) = e^x + \lambda \int_0^1 2e^x e^t\, y(t)dt$. 5+3=8

6. (a) With the aid of the resolvent Kernel, find the solution of the integral equation

$y(x) = e^{x^2} + \int_0^x e^{x^2-t^2}\, y(t)dt$.

(b) Define Bessel's functions of the first kind of order *n*. 6+2=8

7. Define Laguerre's equation and then find its solution. 1+7=8

8. (a) Solve the integral equation $f(x) = \int_a^x \frac{y(t)dt}{(\cos t - \cos x)^{\frac{1}{2}}}$, $0 \le a < x < b \le \pi$.

(b) Using the Rodrigue's formula for $H_n(x)$ and integrating by parts interatively, show that

$$\Psi = \int_{-\infty}^{\infty} \exp(-x^2)\, H_n(x)\, H_m(x) dx = \begin{cases} 0 \quad , & \text{if } m \neq n \\ 2^n \cdot n! \sqrt{\pi}, & \text{if } m = n \end{cases}$$

4+4=8