M.Sc.-I/Math-103C/18

M.Sc. 1st Semester Examination, 2018

MATHEMATICS

Paper : 103C (Topology)

Course ID : 12153

Time: 2 Hours **Full Marks: 40**

The questions are of equal value.
The figures in the right hand side margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meanings.

Answer *any five* questions. 8×5=40

1. (a) Define neighbourhood operator on a non-empty set *X*.

(b) If (X,τ) be a topological space, then show that the mapping $f: X \to P\big(P(X)\big)$, defined by $f(x) = \beta_x, \forall\, x \in X$ where β_x is the system of all neighbourhood of *x*, is a nbd operator on *X*.

(c) Is the union of two topologies on a non-empty set *X* a topology on *X*. 2+4+2=8

2. (a) Show that a collection B of open sets of a topological space (X,τ) forms an open base of τ *iff* $\forall\, U \in \tau$ and $\forall\, p \in U, \exists V_p \in B$ such that $p \in V_p \subseteq U$.

(b) Show that for any two subsets *A*, *B* of a topological space (X,τ), $(A \cup B)' = A' \cup B'$ where A' is the derived set of *A*.

(c) Let (X,τ) be a topological space and $Y \subseteq X$. Show that a point $p \in Y$ is a limit point of $P \subseteq Y$ in (X,τ) if p is a limit point of *P* in (Y,τ_Y). 3+3+2=8

3. (a) Let (X,τ) and (Y,τ') be two topological spaces. Then show that the mapping $f:(X,\tau) \to (Y,\tau')$ is open *iff* $f(A^\circ) \subseteq [f(A)]^\circ, \forall\, A \subseteq X$ where A° is the set of all interior points of *A*.

(b) If $f:(X,\tau) \to (Y,\tau')$ is bijective, closed and continuous, then show that f is a homeomorphism.

(c) Give an example of a mapping which is closed but not open and continuous. 3+2+3=8

4. (a) Let (X,τ) be a T_1-space and $A \subseteq X$. If a point $p \in X$ is a limit point of A, then show that every open set containing p contains an infinite number of points of *A* other than p.

(b) Prove that $T_2 \Rightarrow T_1 \Rightarrow T_0$ and give examples to justify that $T_0 \nRightarrow T_1 \nRightarrow T_2$. 3+5=8

5. (a) Show that a topological space (X,τ) is regular *iff* $\forall\, x \in X$ and $\forall\, U \in \tau$ such that $x \in U$, $\exists\, V \in \tau$ such that $x \in V$ and $\bar{V} \subseteq U$.

(b) Show that any T_4-space is a Tychonoff space.

(c) Define Separable space. 4+3+1=8

6. (a) Prove that continuous image of a compact set is compact.

(b) Show that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.

(c) Give an example of a topological space which is locally compact but not compact. 3+3+2=8

7. (a) Prove that the union of a collection of connected sets no two members of which are separated, is connected.

(b) Show that a topological space (X, τ) is not connected iff $\exists$ a continuous mapping of (X, τ) onto a discrete two point space. 4+4=8

8. (a) If (X, τ) and (Y, τ') be two topological spaces, define the product topology $\tau \times \tau'$ on $X \times Y$. Prove that the projection mapping $p_X: (X \times Y, \tau \times \tau') \to (X, \tau)$: $p_X(x, y) = x, (x, y) \in X \times Y$ and $p_Y: (X \times Y, \tau \times \tau') \to (Y, \tau')$: $p_Y(x, y) = y, (x, y) \in X \times Y$ are continuous and open.

(b) Prove that product of two connected spaces is connected. (2+3)+3=8

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