M.Sc.-I/Math-101C/18

M.Sc. 1st Semester Examination, 2018

MATHEMATICS

Paper : 101C (Abstract Algebra)

Course ID : 12151

Time: 2 Hours **Full Marks: 40**

The questions are of equal value.

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Answer *any five* questions. 8×5=40

1. (a) Let G be a group and A, B be two normal subgroups of G. Then $A \simeq B$ if and only if $\frac{G}{A} \simeq \frac{G}{B}$.
— True or false? Justify.

(b) Find all the subgroups of the group $\frac{\mathbb{Z}}{28\,\mathbb{Z}}$.

(c) Show that for any group G, $\frac{G}{Z(G)} \simeq I_{nn}(G)$. 3+2+3=8

2. (a) State and prove Burnside theorem for group action.

(b) Let G be a group of order $2m$, where m is an odd integer. Show that G has a normal subgroup of order m. (1+3)+4=8

3. (a) Show that the converse of Lagrange's theorem is true for any finite abelian group.

(b) Prove that any group of order 48 is simple. 4+4=8

4. (a) Find $Z(D_4)$.

(b) Give an example of

(i) a subnormal series which is not normal.

(ii) a normal series which is not composition.

(c) Is the group S_3 solvable? Justify.

(d) Prove that every nilpotent group is solvable. 2+3+1+2=8

5. (a) Let R be a ring with identity. Then show that Char $R = n$ if and only if n is the least positive integer such that $n.1 = 0$.

(b) Show that the characteristic of an integral domain is either prime or zero.

(c) Let $(G, +)$ be a simple abelian group. Prove that the ring End G is a division ring. 2+3+3=8

6. (a) Let R be a ring and $S \subseteq R$ be non-empty. Then show that $\{a \in R \mid ax = 0 \ \forall \ x \in S\}$ is a left ideal of R.

(b) Let R be a commutative ring with identity. Then prove that every maximal ideal of R is prime.
Is the converse of the above result true?
Is the above result true in a ring without identity? Justify your answer. 2+(3+2+1)=8

7. (a) Show that a proper ideal I of a ring R is a maximal ideal if and only if for any ideal A of R, either $A \subseteq I$ or $A + I = R$.

(b) Let p be a non-zero non-unit element in an integral domain R. If p is prime then show that p is irreducible.
Show that the converse of above result is not true.
State a condition so that the converse becomes true.

(c) Give an example of a Euclidean domain which is not field. 2+(2+1+1)+2=8

8. (a) Let R be a commutative ring with identity. Then show that R is an integral domain if and only if so is $R[x]$.

(b) State Eisenstein's criterion for irreducibility of a polynomial in $\mathbb{Z}[x]$ over Q.

(c) Let M be an R-module. Then prove that M is Artinian if and only if every non-empty set of submodules of M contains a minimal element under inclusion. 3+1+4=8