SH-I/PHS/101/C-1/19

B.Sc. 1st Semester (Honours) Examination, 2019-20

PHYSICS

Course ID : 12411 **Course Code : SHPHS/101/C-1**

Course Title : Mathematical Physics-I

Time: 1 Hour 15 Minutes **Full Marks: 25**

The figures in the margin indicate full marks.

Section-I

1. Answer *any five* questions: 1×5=5

(a) If $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ prove that $\Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi}$.

(b) Find the Fourier Co-efficient as for half wave rectifier.

(c) Find the directional derivative of a scalar function $\varphi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.

(d) Show that $\vec{E} = \frac{\vec{r}}{r^2}$ is irrotational.

(e) Write down the expression for the elements of area in cylindrical co-ordinate (ρ, φ, z) system for $\rho =$ constant surface.

(f) Suppose $\vec{\nabla} \cdot \vec{\beta} = 0$. Comment about the nature of vector field.

(g) Write the error function.

(h) If $\vec{A}$ is a constant vector, prove that $\vec{\nabla}(\vec{r} \cdot \vec{A}) = \vec{A}$.

Section-II

Answer *any two* questions. 5×2=10

2. Find the Fourier series of the following function $f(x) = \begin{cases} -m & when \quad -\pi < x < 0 \\ m & when \quad 0 < x < \pi \end{cases}$.

Hence, show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \ldots$ 4+1=5

3. Using the method of separation variable arrive at the radial equation

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} - \frac{l(l+1)R}{r^2} = 0$$

from the Laplace equation $(\nabla^2\phi = 0)$ in spherical polar co-ordinate system and show that the general solution of radial equation is of the form

$R(r) = Ar^l + Br^{-(l+1)}$, where A and B are constant.

4. (a) Define Γ(gamma) function in integral form.

(b) What is the domain of convergence of this integral form?

(c) Find the relation $\Gamma(n) = (n-1)!$. 2+1+2=5

5. (a) State Green's theorem in the plane.

(b) Evaluate the integral $\int_c (xy + y^2)dx + x^2dy$, where *c* is the closed curve of the region bounded by $y = x$ and $y = x^2$ and verify Green's theorem in the plane. 1+4=5

Section-III

6. Answer *any one* question: 10×1=10

Consider the differential equation $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$

(a) Find singular point and justify regular or irregular singular point.

(b) Find the roots of the indicial equation.

(c) Prove the recurrence relation $J_{n(x)} = \frac{x}{2n}\left[J_{n-1(x)} + J_{n+1(x)}\right]$. 3+3+4=10

7. (a) Find a unit rector parallel to the *xy* plane and perpendicular to the vector $4\hat{\imath} - 3\hat{\jmath} + \hat{k}$.

(b) Prove that, $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$.

(c) Prove $(\vec{v} \cdot \vec{\nabla})\vec{v} = \frac{1}{2}\vec{\nabla}v^2 - \vec{v} \times (\vec{\nabla} \times \vec{v})$. 2+3+5=10

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