B.Sc. 1st Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 12114 **Course Code : SH/MTH/103/GE-1**

Course Title : Calculus, Geometry and Differential Equation

Time 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Obtain the perimeter of the circle $x^2 + y^2 = a^2$.

(b) Find $\int_0^{\pi/2} \sin^6 x \cos^8 x . dx$.

(c) Find $\lim_{t\to\propto} x(t)$ where $x(t)$ satisfies the differential equation $\dot{x} + x = 0$ with $x(0) = 2$. where $\dot{x} = \frac{dx}{dt}$.

(d) Find the general solution of $\frac{dy}{dx} + Ay = B$, where *A* and *B* are functions of *x* alone.

(e) Solve the equation $y = px + \sqrt{a^2p^2 + b^2}$ and obtain the singular solution.

(f) Find the centre and radius of the sphere $2(x^2 + y^2 + z^2) - 2x + 4y - 6z = 15$.

(g) Find the asymptotes of $xy - 3x - 4y = 0$.

(h) Find the equation of the line $y = \sqrt{3}\, x$ when the axes are rotated through an angle $\frac{\pi}{3}$.

2. Answer *any four* questions: 5×4=20

(a) If $y = e^{a \sin^{-1} x}$, then show that

(i) $(1 - x^2)y_2 - xy_1 - a^2y = 0$,

(ii) $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. 2+3=5

(b) Establish reduction formula for $\int \sin^n x \, dx$ and evaluate $\int_0^{\pi/2} \sin^5 x \, dx$. 3+2=5

(c) Solve the equation $(y^2e^x + 2xy)dx - x^2dy = 0$.

(d) Find the enveloping cone of the ellipsoid $x^2 + 3y^2 + 5z^2 = 1$ with its vertex at (1, 2, 3).

(e) Reduce the equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$ to its canonical form and determine the nature of the conic represented by it.

(f) (i) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta), y = a(1 + \cos\theta)$ about its base.

(ii) Show that the semi-latus rectum of a conic is a harmonic mean between the segments of any focal chord. 3+2=5

3. Answer *any one* question: 10×1=10

(a) (i) If $I_n = \int_0^1 x^n \tan^{-1} x.dx$, then show that $(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$.

(ii) A body whose temperature is initially 100°C is allowed to cool in air whose temperature remains at a constant temperature 20°C. It is given that after 10 minutes, the body has cooled to 40°C. Find the temperature of the body after 30 minutes.

(iii) Show that the conic $\frac{l_1}{r} = 1 - e_1 \cos\theta$ and $\frac{l_2}{r} = 1 - e_2 \cos(\theta - \alpha)$ will touch each other if $l_1^2(1 - e_2^2) + l_2^2(1 - e_1^2) = 2l_1l_2(1 - e_1e_2\cos\alpha\ .)$ 3+4+3=10

(b) (i) Show that the total arc length of the ellipse $x = a\cos t, y = b\sin t, 0 \le t \le 2\pi$ for $a > b > 0$ is given by $4a\int_0^{\pi/2}\sqrt{1 - K^2\cos^2 t}\, dt$, where $k = \frac{\sqrt{a^2-b^2}}{a}$.

(ii) Solve : $(4x^2y - 6)dx + x^3dy = 0$ 5+5=10

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