B.Sc. Semester I (Honours) Examination, 2018-19
MATHEMATICS

Course Id : 12114 **Course Code : SHMTH-103GE-1(T)**

Course Title : Calculus, Geometry & Differential Equation

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Find the *n*-th derivative of the function $y = \log(x + a)$.

(b) Solve: $(x + y + 1)dy = dx$

(c) Evaluate: $\int_0^{\frac{\pi}{2}} \cos^4 x \sin^2 x dx$

(d) Find the envelope of the straight line $y = mx + \frac{a}{m}, m$ being the variable parameter $(m \neq 0)$.

(e) Reduce the differential equation $xy' + y = y^2 \log x$ to a linear form.

(f) Find the asymptotes of $x^2 - 4y^2 = 1$.

(g) Find the equation of the sphere through the points (0, 0, 0), (*a*, 0, 0), (0, *b*, 0) and (0, 0, *c*).

(h) Transform the equation $x^2 + 2\sqrt{3}xy - y^2 - 2 = 0$ to axes inclined at 30° to the original axes.

2. Answer *any four* questions: 5×4=20

(a) (i) Find the value of $\lim_{x \to 1} \left(x^{\frac{1}{1-x}} \right)$.

(ii) If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ is finite, find a and the value of the limit. 2+(2+1)=5

(b) If $I_n = \int \sec^n x dx$, then show that $(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$ and hence evaluate $\int_0^{\frac{\pi}{4}} \sec^5 x dx$. 3+2=5

(c) (i) Find the order and degree of the differential equation $\sqrt{y + \left(\frac{dy}{dx}\right)^2} = 1 + x$.

(ii) Find the general solution of the differential equation $\frac{dy}{dx} = (x^2 + 1)(y^2 + 1)$. 2+3=5

(d) If $y = \cos(10 \cos^{-1} x)$, show that $(1 - x^2)y_{12} = 21xy_{11}$. State Leibnitz's theorem. 4+1=5

(e) Obtain singular solution of the equation $y = px + p - p^2$, where $p = \frac{dy}{dx}$. 5

(f) The radius of a right circular cylinder is 2 and its axis is given by $\frac{x}{1} = \frac{y}{-2} = \frac{z}{2}$; Find the equation of the cylinder. 5

3. Answer *any one* question: 10×1=10

(a) (i) Find the equation of the cone whose vertex is at (1, 0, –1) and which passes through the circle $x^2 + y^2 + z^2 = 4, x + y + z = 1$.

(ii) Find the length of the astroid $x = \cos^3 t, y = \sin^3 t, 0 \leq t \leq 2\pi$.

(iii) Using L'Hospital Rule, evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$. 4+3+3=10

(b) (i) Solve: $(1 + xy)ydx + (1 - xy)xdy = 0$

(ii) Show that in any conic the sum of the reciprocals of the segments of a focal chord is constant. 5+5=10

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