B.SC. FIRST SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 12114**

Course Code: SH/MTH/103/GE-1

Course Title: Calculus, Geometry & Differential Equations

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

1. **Answer *any five* of the following questions:** $(\mathbf{2 \times 5 = 10})$

a) Show that the curve $y = x^3$ has a point of inflection at $x = 0$.

b) Calculate $\int_0^{\frac{\pi}{2}} \sin^7 x\, dx$

c) Find the nature of the conic $9x^2 + 24xy + 16y^2 - 126x + 82y - 59 = 0$.

d) Find the envelope of the family of the straight lines $y = mx + \sqrt{a^2m^2 + b^2}$, where m being a parameter.

e) Solve:$(x^2 - y^2)dy = xydx$.

f) Find the value of m for which the plane $x + y + z = m$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0.$$

g) Find singular solution of $y = px + p - p^2$.

h) Show that the differential equation $(x^3 - 3x^2y + 2xy^2)dx - (x^3 - 2x^2y + y^2)dy = 0$ is exact

2. **Answer *any four* of the following questions:** $(\mathbf{5 \times 4 = 20})$

a) If $I_{m,n} = \int \sin^{\mathrm{m}} x \cos^{\mathrm{n}} x\, dx$ then prove that

$$I_{m,n} = \frac{\sin^{\mathrm{m+1}} x \cos^{\mathrm{n-1}} x}{m+n} + \frac{\mathrm{n-1}}{\mathrm{m+n}} I_{m,n-2}.$$

b) A sphere of constant radius $2a$ passes through origin meets the axes in A, B, C. Show that the locus of a centroid of the tetrahedron $OABC$ is the sphere $x^2 + y^2 + z^2 = a^2$.

c) Find the asymptotes of $x^3 + y^3 = 3axy$.

d) (i) Find the equation of the cone whose vertex is the $(1,2,3)$ and the guiding curve is the circle $x^2 + y^2 + z^2 = 4,\quad x + y + z = 1$.

(ii) Find the equation of the sphere which passes through the circle

$x^2 + y^2 + z^2 = 9,\quad 2x + 3y + 4z = 5$ and the point $(1,2,3)$. 3+2

e) State Leibnitz Rule for n^{th} order derivative. If $y = x^n \log x$, then prove that

$y_n = n!\left[\log x + 1 + \frac{1}{2} + \cdots + \frac{1}{n}\right]$. 1+4

f) (i) Prove that the necessary and sufficient condition for the ODE $Mdx + Ndy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(ii) Solve: $(x^4 + y^4)dx - xy^3 dy = 0$. 3+2

3. Answer *any one* of the following questions: $(\mathbf{10 \times 1 = 10})$

a) (i) Solve $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$

(ii) Find the length of the perimeter of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

(iii) Find the nature of the conic $\frac{8}{r} = 4 - 5\cos\theta$ 4+4+2

b) (i) Find the area of the segment of the parabola $y = (x-1)(x-4)$cutoff by the x-axis.

(ii) Solve: $(e^x + 1)ydy = (y^2 + 1)e^x dx$, given $y = 0$ when$x = 0$.

(iii) Examine the concavity and convexity about x-axis for the curve $x = 6t^2$, $y = 4t^3 - 3t$. 3+4+3

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