SH-I/Mathematics-102/C-2/19

B.Sc. 1st Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 12112 **Course Code : SH/MTH/102/C-2**

Course Title : Algebra

Time 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Is union of two equivalence relations an equivalence relation? (Justify).

(b) If x, y, z are three positive real numbers, show that $\frac{yz}{x} + \frac{xz}{y} + \frac{xy}{z} \geq x + y + z$.

(c) If A be an invertible matrix, then show that A^{-1} is invertible and $(A^{-1})^{-1} = A$.

(d) A matrix A has eigenvalues 1 and 4 with corresponding eigenvectors $(1, -1)^T$ and $(2, 1)^T$ respectively. Find the matrix A.

(e) Prove that $\sqrt[n]{i} + \sqrt[n]{-i} = 2cos\frac{\pi}{2n}$.

(f) Find the remainder when 4^{119} is divided by 9.

(g) V and W are two subspaces of R^n and $T : V \rightarrow W$ is a linear transformation. Prove that $T(\theta_v) = \theta_w$ where the symbols have the usual meaning. Hence show that

$T(-\alpha) = -T(\alpha)\ \forall\ \alpha\ \in\ V$.

(h) If α, β, γ are the roots of $x^3 - px^2 + qx - r = 0$, obtain the value of $\sum\frac{1}{\alpha^2}$.

2. Answer *any four* questions: 5×4=20

(a) (i) A relation ρ on Z, (Z, be the set of integers), such that $x\rho y$ if and only if $4x + 7y$ is divisible 11, for $x, y\ \in\ Z$. Verify the relation ρ is an equivalence relation on Z or not.

(ii) $n(> 1)$ be a positive integer then show that $(n+1)^{n-1}\ \ (n+2)^n > 3^n(n!)^2$. 3+2=5

(b) (i) Define cardinality of a set. Do the sets *Z* and *N* have same cardinal number? (*Z* and *N* be the set of integers and natural numbers.)

(ii) If α be a root of the equation $x^7 = 1$, then show that the roots of the equation.

$x^2 + x + 2 = 0$ are $(\alpha + \alpha^2 + \alpha^4)$ and $(\alpha^3 + \alpha^5 + \alpha^6)$. (1+2)+2=5

(c) (i) Find the greatest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

(ii) Using mathematical induction, prove that there are 2^n subsets of a set of *n* elements.
3+2=5

(d) Let $f: R \to R$ be a mapping defined by

$f(x) = |\,x\,| + x, x \in R$ and

$g : R \to R$ be another mapping defined by $g(x) = |\,x\,| - x,\ x \in R$;

Find the compositions $g \circ f$ and $f \circ g$. 5

(e) Determine the linear mapping $T : R^3 \to R^3$ that maps the basis vectors (0, 1, 1), (1, 0, 1), (1, 1, 0) of R^3 to (2, 1, 1), (1, 2, 1), (1, 1, 2) respectively. 5

(f) The eigenvalues of a 3 × 3 matrix *A* are in A.P. and given that |*A*| = 80, Trace *A* = 15. Find the eigenvalues. 5

3. Answer *any one* question: 10×1=10

(a) (i) Examine if the relation ρ defined on Z by $\rho = \{(a, b) \in Z \times Z : 7/3a + 4b\}$ is an equivalence relation.

(ii) Verify Caley-Hamilton theorem for the square matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \text{ Hence find } A^{-1}.$$

(iii) From the result of 3a (ii) above, compute A^{100}. 3+4+3=10

(b) (i) If $cl(a)$ is an equivalence class and $b \in cl(a)$ then prove that $cl(a) = cl(b)$.

(ii) Find $g \circ f$ and $f \circ g$ if $f : R \to R$ be defined by $f(x) = |x| + x, x \in R$ g: and $R \to R$ be defined by $g(x) = |\,x\,| - x,\ x \in R$.

(iii) Determine the condition for which the following system of equation has

(I) only one solution (II) no solution (III) many solution

$$x + y + z = b$$
$$2x + y + 3z = b + 1$$
$$5x + 2y + az = b^2$$

2+3+5=10

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