B.Sc. Semester I (Honours) Examination, 2018-19
MATHEMATICS

Course Id : 12112 **Course Code : SHMTH-102C-2(T)**

Course Title : Algebra

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Use De Moivre's theorem to prove that $\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$.

(b) State Division algorithm. Use it to find the remainder when – 326 is divided by 5. 1+1=2

(c) If $S = a + b + c$, prove that $\frac{s}{a-b} + \frac{s}{s-b} + \frac{s}{s-c} > {}^9/_2$.

(d) Find all the equivalence relations on the set $S = \{a, b, c\}$.

(e) For what values of k, the following system of equations has a non-trivial solution?

$$x + 2y + 3z = kx, 2x + y + 3z = ky, 2x + 3y + z = kz$$

(f) When a transformation $T: R^m \to R^n$ is said to be a linear transformation? Is the following transformation linear?

$T: R^2 \to R$ defined by

$T(x, y) = |x - y|$, for all $x, y \in R$.

(g) If an equation $f(x) = 0$ with real coefficients consists of only even powers of x with all positive signs, show with proper reason that the equation cannot have a real root.

(h) Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 2 \end{pmatrix}$. Find the eigenvalues of the matrix $A^5 - I_3$.

2. Answer *any four* questions: 5×4=20

(a) Define rank of a matrix. Find the row reduced echelon form of the matrix

$\begin{pmatrix} 1 & -1 & 2 & 0 & 4 \\ 2 & 2 & 1 & 5 & 2 \\ 1 & 3 & -1 & 0 & 3 \\ 1 & 7 & -4 & 1 & 1 \end{pmatrix}$ and find its rank. 1+3+1=5

(b) (i) If $ax \equiv ay (mod\ n)$ and a is prime to n, then show that $x \equiv y (mod\ n)$.

(ii) Obtain the equation whose roots are the roots of the equation $x^4 - 8x^2 + 8x + 6 = 0$, each diminished by 2. 1+4=5

(c) If a, b, c be positive real numbers such that the sum of any two is greater than the third, then prove that $abc \geq (a + b - c)(b + c - a)(c + a - b)$. Derive when equality occurs. 4+1=5

(d) State the second principle of mathematical induction and using this principle, prove that $(3+\sqrt{5})^n + (3-\sqrt{5})^n$ is divisible by 2^n, for all $n \in N$ (*N* be set of natural numbers). 5

(e) Show that the eigenvalues of a real symmetric matrix are all real. 5

(f) (i) Show that $A = \{(x_1, x_2, x_3, \dots x_n) \in R^n : x_1 + x_2 + \cdots + x_n = 0\}$ is a subspace of R^n. Find the dimension of the subspace A of R^n.

(ii) Is the union of two subspaces of R^n, a subspace of R^n? Justify your answer. 3+2=5

3. Answer *any one* question: 10×1=10

(a) (i) Show that the transformation $T: R^3 \to R^2$ defined by $T(x, y, z) = (z, x + y)$ is linear.

(ii) Let $f: A \to B$ and $g: B \to C$ be two mappings such that $g_\circ f: A \to C$ is injective. Is it necessary that g is injective? Justify your answer.

(iii) Solve the cubic equation $x^3 - 9x + 28 = 0$ by Cardon's method. 3+2+5=10

(b) (i) Prove that any partition of a non-empty set S induces an equivalence relation on S.

(ii) Prove or disprove:

"Composition of two linear transformation from $R^m \to R^n$ is linear transformation."

(iii) A linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x_1, x_2, x_3) = (2x_1 + x_2 - x_3, x_2 + 4x_3, x_1 - x_2 + 3x_3)(x_1, x_2, x_3) \in R^3$. Obtain the matrix of *T* relative to the ordered base (0, 1, 1), (1, 0, 1), (1, 1, 0) of $\mathbb{R}^3$. 3+2+5=10
