SH-I/Mathematic-101/C-I/19

B.Sc. 1st Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 12111 **Course Code : SH/MTH/101/C-1**

Course Title : Calculus, Geometry and Differential Equations

Time 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Obtain a reduction formula for $\int x^n e^{-ax} dx, (n \neq -1)$.

(b) Find the equation of the sphere whose centre is at (1, 2, 3) and which passes through the point (7, 8, 9).

(c) Evaluate: $\lim_{x \to \pi/2} (1 - \sin x) \tan x$.

(d) Find the envelope of the curve $x^2 \cos\theta + y^2 \sin\theta = a^2$, θ is a parameter.

(e) Find the equation of the directrix of the conic $r \sin^2 \frac{\theta}{2} = 1$.

(f) Obtain the asymptotes of the given curve $xy = 25$.

(g) Solve: $x^2 y dx - (x^3 + y^3) dy = 0$.

(h) Find the value of m for which the plane $x + y + z = m$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$.

2. Answer *any four* questions: 5×4=20

(a) Reduce the equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$ to its standard form and show that it represents a hyperbola. 5

(b) (i) Find the value of the constants a and b such that $\lim_{x \to 0} \frac{x(1 + a\cos x) - b \sin x}{x^3} = 1$.

(ii) If $y = \sin^{-1} x$, then show that $(1 - x^2) y_2 - x y_1 = 0$. 3+2=5

(c) Show that the valume of the solid obtained by revolving the cardiode $r = a(1 + \cos\theta)$ about the initial line is $\frac{8}{3}\pi a^3$. 5

(d) Find the condition that the straight line $\frac{l}{r} = a\cos\theta + b\sin\theta$ may touch the circle $r = 2d\cos\theta$.

(e) Determine the asymptotes of the curve $(x - y)(x + y)(x + 2y) + y(x - y) + 1 = 0$. 5

(f) (i) Prove that the number of integrating factors of an equation $Mdx + Ndy = 0$, which has a solution, is infinite.

(ii) Solve: $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$. 3+2=5

3. Answer *any one* : 10×1=10

(a) (i) Show that $\lim_{\theta \to \frac{\pi}{2}} (\cos\theta)^{\cos\theta} = 1$

(ii) Find the point of inflexion of the curve $a^2 = r^2\theta$.

(iii) The circle $x^2 + y^2 = a^2$ is devided by the hyperbola $x^2 - 2y^2 = \frac{a^2}{4}$. Find out the area of the portion of the circle which is not contained in the hyperbola. 3+3+4=10

(b) (i) Prove that the necessary and sufficient condition for ODE $Mdx + Ndy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(ii) Find the point of inflexion, if any, of the curve $y(a^2 + x^2) = x^3$.

(iii) If $y = (x^2 - 1)^n$, then prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} = n(n + 1)y_n$. 4+3+3=10

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