B.Sc. Semester I (Honours) Examination, 2018-19
MATHEMATICS

Course Id : 12111 **Course Code : SHMTH-101C-1(T)**

Course Title : Calculus, Geometry and Differential Equation

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Evaluate: $\lim_{x\to 1} \frac{\log(1-x)}{\cot(\pi x)}$

(b) Define rectilinear asymptotes of a plane curve.

(c) Find the perimeter of the cardioid $r = a(1 - \cos\theta)$

(d) Define Bernoulli's equation. Can it be put in the linear form? Justify your answer.

(e) Solve : $\frac{dy}{dx} + y\frac{d\Phi(x)}{dx} = \Phi(x)\frac{d\Phi(x)}{dx}$

(f) Show that the semi-latus rectum of a conic is a harmonic mean between the segments of any focal choral.

(g) Find the nature of the conicoid: $3x^2 - 2y^2 - 12x - 12y - 6z = 0$.

(h) Prove that $(a-2, {}^{-2}/_{e^2})$ is a point of inflexion of the curve $y = (x-a)e^{x-a}$.

2. Answer *any four* questions: 5×4=20

(a) If $J_n = \int \sec^n x\, dx$, then show that $(n-1)J_n = \tan x\, sec_x^{n-2} + (n-2)J_{n-2}$. Hence, find a reduction formula for $\int_0^{\frac{\pi}{4}} \sec^n x\, dx$ and use this formula to evaluate $\int_0^{\frac{\pi}{4}} \sec^5 x\, dx$. 2+1+2=5

(b) Define envelope of a family of curves. Find the condition between a and b so that the envelope of the family of the lines ${}^x/_a + {}^y/_b = 1$ may be the curve $x^p y^q = k^{p+q}$. 2+3=5

(c) (i) Show that if a straight line meets a conicoid in three points, then the straight line lies wholly on the conicoid.

(ii) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. 2+3=5

(d) Find the area common to the circles $r = a\sqrt{2}$ and $r = 2a\cos\theta$. 5

(e) What do you mean by Integrating Factor? Find the integrating factor of the ODE $(xy\sin xy + \cos xy)ydx + (xy\sin xy - \cos xy)xdy = 0$. Hence solve it. 1+1+3=5

(f) If the population of a country doubles in 100 years, in how many years will it treble under the assumption that the rate of increase is proportional to the number of inhabitants? 5

3. Answer *any one* question: 10×1=10

(a) (i) If $y = \frac{\log x}{x}$, prove that $y_n = \frac{d^n y}{dx^n} = \frac{(-1)^n \lfloor n}{x^{n+1}} \left[\log x - 1 - \frac{1}{2} - \frac{1}{3} \dots - \frac{1}{n}\right]$.

(ii) Find the asymptotes of the curve

$$\left.\begin{aligned} x &= \frac{1}{t^4 - 1} \\ y &= \frac{t^3}{t^4 - 1} \end{aligned}\right\}$$

(iii) Find the surface generated by the revolution of an arc of the catenary $y = c \cosh {}^{x}/_{c}$ about the axis of x. 3+3+4=10

(b) (i) Find the values of b and g such that the equation $9x^2 + 12xy + by^2 + 2gx + 4y + 1 = 0$ represents a conic having infinitely many centres and determine the nature of the conic.

(ii) A variable plane is parallel to the given plane ${}^{x}/_{a} + {}^{y}/_{b} + {}^{z}/_{c} = 2$ and meets the axes in A, B, C respectively. Show that the circle ABC lies on the cone

$$\left({}^{b}/_{c} + {}^{c}/_{b}\right)yz + \left({}^{a}/_{c} + {}^{c}/_{a}\right)zx + \left({}^{a}/_{b} + {}^{b}/_{a}\right)xy = 0$$

(iii) Show that if y_1 and y_2 are two solutions of the ODE $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x only and $y_2 = y_1 z$ then show that $z = 1 + ae^{-\int \frac{Q}{y_1} dx}$, a being an arbitrary constant. 3+4+3=10
