B.Sc. 1st Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 12112 **Course Code : SH/MTH/102/C-2**

Course Title : Algebra

Time 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Is union of two equivalence relations an equivalence relation? (Justify).

(b) If x, y, z are three positive real numbers, show that $\frac{yz}{x}+\frac{xz}{y}+\frac{xy}{z} \geq x+y+z$.

(c) If A be an invertible matrix, then show that A^{-1} is invertible and $(A^{-1})^{-1}=A$.

(d) A matrix A has eigenvalues 1 and 4 with corresponding eigenvectors $(1,-1)^T$ and $(2,1)^T$ respectively. Find the matrix A.

(e) Prove that $\sqrt[n]{i}+\sqrt[n]{-i}=2cos\frac{\pi}{2n}$.

(f) Find the remainder when 4^{119} is divided by 9.

(g) V and W are two subspaces of R^n and $T : V \rightarrow W$ is a linear transformation. Prove that $T(\theta_v)=\theta_W$ where the symbols have the usual meaning. Hence show that

$T(-\alpha)=-T(\alpha)\ \forall\ \alpha\ \in\ V$.

(h) If α, β, γ are the roots of $x^3-px^2+qx-r=0$, obtain the value of $\sum\frac{1}{\alpha^2}$.

2. Answer *any four* questions: 5×4=20

(a) (i) A relation ρ on Z, (Z, be the set of integers), such that $x\rho y$ if and only if $4x+7y$ is divisible 11, for $x, y\ \in\ Z$. Verify the relation ρ is an equivalence relation on Z or not.

(ii) $n(>1)$ be a positive integer then show that $(n+1)^{n-1}\ (n+2)^n > 3^n(n!)^2$. 3+2=5

(b) (i) Define cardinality of a set. Do the sets Z and N have same cardinal number? (Z and N be the set of integers and natural numbers.)

(ii) If α be a root of the equation $x^7=1$, then show that the roots of the equation.

$x^2+x+2=0$ are $(\alpha+\alpha^2+\alpha^4)$ and $(\alpha^3+\alpha^5+\alpha^6)$. (1+2)+2=5

(c) (i) Find the greatest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

(ii) Using mathematical induction, prove that there are 2^n subsets of a set of *n* elements. 3+2=5

(d) Let $f: R \to R$ be a mapping defined by

$f(x) = |\,x\,| + x, x \in R$ and

$g : R \to R$ be another mapping defined by $g(x) = |\,x\,| - x,\ x \in R$;

Find the compositions $g \circ f$ and $f \circ g$. 5

(e) Determine the linear mapping $T : R^3 \to R^3$ that maps the basis vectors (0, 1, 1), (1, 0, 1), (1, 1, 0) of R^3 to (2, 1, 1), (1, 2, 1), (1, 1, 2) respectively. 5

(f) The eigenvalues of a 3 × 3 matrix *A* are in A.P. and given that |*A*| = 80, Trace *A* = 15. Find the eigenvalues. 5

3. Answer *any one* question: 10×1=10

(a) (i) Examine if the relation ρ defined on Z by $\rho = \{(a, b) \in Z \times Z : 7/3a + 4b\}$ is an equivalence relation.

(ii) Verify Caley-Hamilton theorem for the square matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$ Hence find A^{-1}.

(iii) From the result of 3a (ii) above, compute A^{100}. 3+4+3=10

(b) (i) If $cl(a)$ is an equivalence class and $b \in cl(a)$ then prove that $cl(a) = cl(b)$.

(ii) Find $g \circ f$ and $f \circ g$ if $f : R \to R$ be defined by $f(x) = |x| + x, x \in R$ g: and $R \to R$ be defined by $g(x) = |\,x\,| - x,\ x \in R$.

(iii) Determine the condition for which the following system of equation has

(I) only one solution (II) no solution (III) many solution

$$x + y + z = b$$
$$2x + y + 3z = b + 1$$
$$5x + 2y + az = b^2$$

2+3+5=10

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B.Sc. 1st Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 12114 **Course Code : SH/MTH/103/GE-1**

Course Title : Calculus, Geometry and Differential Equation

Time 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Obtain the perimeter of the circle $x^2 + y^2 = a^2$.

(b) Find $\int_0^{\pi/2} \sin^6 x \cos^8 x . dx$.

(c) Find $\lim_{t \to \propto} x(t)$ where $x(t)$ satisfies the differential equation $\dot{x} + x = 0$ with $x(0) = 2$. where $\dot{x} = \frac{dx}{dt}$.

(d) Find the general solution of $\frac{dy}{dx} + Ay = B$, where *A* and *B* are functions of *x* alone.

(e) Solve the equation $y = px + \sqrt{a^2p^2 + b^2}$ and obtain the singular solution.

(f) Find the centre and radius of the sphere $2(x^2 + y^2 + z^2) - 2x + 4y - 6z = 15$.

(g) Find the asymptotes of $xy - 3x - 4y = 0$.

(h) Find the equation of the line $y = \sqrt{3}\, x$ when the axes are rotated through an angle $\frac{\pi}{3}$.

2. Answer *any four* questions: 5×4=20

(a) If $y = e^{a \sin^{-1} x}$, then show that

(i) $(1 - x^2)y_2 - xy_1 - a^2y = 0$,

(ii) $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. 2+3=5

(b) Establish reduction formula for $\int \sin^n x\, dx$ and evaluate $\int_0^{\pi/2} \sin^5 x\, dx$. 3+2=5

(c) Solve the equation $(y^2e^x + 2xy)dx - x^2dy = 0$.

(d) Find the enveloping cone of the ellipsoid $x^2 + 3y^2 + 5z^2 = 1$ with its vertex at (1, 2, 3).

(e) Reduce the equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$ to its canonical form and determine the nature of the conic represented by it.

(f) (i) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta), y = a(1 + \cos\theta)$ about its base.

(ii) Show that the semi-latus rectum of a conic is a harmonic mean between the segments of any focal chord. 3+2=5

3. Answer *any one* question: 10×1=10

(a) (i) If $I_n = \int_0^1 x^n \tan^{-1} x.dx$, then show that $(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$.

(ii) A body whose temperature is initially 100°C is allowed to cool in air whose temperature remains at a constant temperature 20°C. It is given that after 10 minutes, the body has cooled to 40°C. Find the temperature of the body after 30 minutes.

(iii) Show that the conic $\frac{l_1}{r} = 1 - e_1 \cos\theta$ and $\frac{l_2}{r} = 1 - e_2 \cos(\theta - \alpha)$ will touch each other if $l_1^2(1 - e_2^2) + l_2^2(1 - e_1^2) = 2l_1 l_2(1 - e_1 e_2 \cos\alpha$.) 3+4+3=10

(b) (i) Show that the total arc length of the ellipse $x = a\cos t, y = b\sin t, 0 \le t \le 2\pi$ for $a > b > 0$ is given by $4a\int_0^{\pi/2} \sqrt{1 - K^2\cos^2 t}\, dt$, where $k = \frac{\sqrt{a^2 - b^2}}{a}$.

(ii) Solve : $(4x^2y - 6)dx + x^3 dy = 0$ 5+5=10

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SP-I/Mathematics-101/C-1A/19

B.Sc. 1st Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 12118 **Course Code : SP/MTH/101/C-1A**

Course Title : Calculus, Geometry and Differential Equations

Time 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Unless otherwise mentioned, notations and symbol is have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Find $\int_0^{\pi/2} \sin^8 x \cos^{10} x \, dx$.

(b) Find $\lim_{x\to 0}(\cos x)^{1/x^2}$.

(c) If the co-ordinate axes are rotated through an angle 45° without changing the origin, find the transformed equation for $x^2 - y^2 = a^2$.

(d) Find the general solution of $\frac{dy}{dx} + Ay = B$ where A, B are function of x alone.

(e) Find an integrating factor of the differential equation $\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \frac{1}{4}(x + xy^2)dy = 0$.

(f) Find the envelope of the family of straight line $x \cos\alpha + y \sin\alpha = a$, α is the parameter.

(g) Find the nature of the conic represented by $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$.

(h) Evaluate $\lim_{x\to 0} \frac{\sin x - x}{x^3}$.

2. Answer *any four* questions: 5×4=20

(a) State Leibnitz's theorem on successive derivatives. If $y = \log(x + \sqrt{1+x^2})$, then show that $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + ny_n = 0$. 1+4=5

(b) Reduce the equation $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$ to its canonical torm and determine the type of the conic represented by it.

(c) Define singular solution of an ordinary differential equation. If y_1 and y_2 be solutions of the equation $\frac{dy}{dx} + P(x)y = Q(x)$ and $y_2 = y_1 Z$, then show that $Z = 1 + a \cdot \bar{e}^{\int (Q/y_1)dx}$, where a is an arbitrary consant. 1+4=5

(d) (i) If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$, then show that $n(I_{n+1} + I_{n-1}) = 1$.

(ii) Show that the semi-latus rectum of a conic is a harmonic mean between the segments of any focal chord. 3+2=5

(e) (i) Solve: $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$.

(ii) Find the envelope of the straight line $y = mx + \frac{a}{m}$, m being a parameter.

(f) Find the asymptotes of $x^3 + 2x^2y + xy^2 - x + 1 = 0$.

3. Answer *any one* question: 10×1=10

(a) (i) If $\lim_{x\to 0} \frac{\sin 2x + a \sin x}{x^3}$ is finite, find a and the value of the limit.

(ii) If $Z_n = \int_0^{\pi/2} x^n \sin x \, dx \ (n \geq 1)$, show that $Z_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)Z_{n-2}$.

(iii) Solve $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$, given that $y = 1$ when $x = 1$. 3+4+3=10

(b) (i) The number of bacteria in a yeast culture grows at a rate proportional to the number present. If the population of a colony yeast bacteria triple in 1 hour, find the number of bacteria that will be present at the end of 5 hours.

(ii) Prove that no two generators of the same system of a hyperboloid of one sheet intersect.

(iii) Show that the straight line $\frac{l}{r} = A\cos Q + B\sin Q$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$ if $(A - e)^2 + B^2 = 1$. 3+4+3=10

SH-I/Mathematic-101/C-I/19

B.Sc. 1st Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 12111 **Course Code : SH/MTH/101/C-1**

Course Title : Calculus, Geometry and Differential Equations

Time 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Obtain a reduction formula for $\int x^n e^{-ax}dx, (n \neq -1)$.

(b) Find the equation of the sphere whose centre is at (1, 2, 3) and which passes through the point (7, 8, 9).

(c) Evaluate: $\lim_{x\to\pi/2}(1-\sin x)\tan x$.

(d) Find the envelope of the curve $x^2\cos\theta + y^2\sin\theta = a^2, \theta$ is a parameter.

(e) Find the equation of the directrix of the conic $r\sin^2\frac{\theta}{2} = 1$.

(f) Obtain the asymptotes of the given curve $xy = 25$.

(g) Solve: $x^2ydx - (x^3+y^3)dy = 0$.

(h) Find the value of m for which the plane $x+y+z=m$ touches the sphere $x^2+y^2+z^2-2x-2y-2z-6=0$.

2. Answer *any four* questions: 5×4=20

(a) Reduce the equation $x^2-5xy+y^2+8x-20y+15=0$ to its standard form and show that it represents a hyperbola. 5

(b) (i) Find the value of the constants a and b such that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3} = 1$.

(ii) If $y = \sin^{-1}x$, then show that $(1-x^2)y_2 - xy_1 = 0$. 3+2=5

(c) Show that the valume of the solid obtained by revolving the cardiode $r = a(1+\cos\theta)$ about the initial line is $\frac{8}{3}\pi a^3$. 5

(d) Find the condition that the straight line $\frac{l}{r} = a\cos\theta + b\sin\theta$ may touch the circle $r = 2d\cos\theta$.

(e) Determine the asymptotes of the curve $(x-y)(x+y)(x+2y)+y(x-y)+1=0$. 5

(f) (i) Prove that the number of integrating factors of an equation $Mdx + Ndy = 0$, which has a solution, is infinite.

(ii) Solve: $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$. 3+2=5

3. Answer *any one* : 10×1=10

(a) (i) Show that $\lim_{\theta \to \frac{\pi}{2}} (\cos\theta)^{\cos\theta} = 1$

(ii) Find the point of inflexion of the curve $a^2 = r^2\theta$.

(iii) The circle $x^2 + y^2 = a^2$ is devided by the hyperbola $x^2 - 2y^2 = \frac{a^2}{4}$. Find out the area of the portion of the circle which is not contained in the hyperbola. 3+3+4=10

(b) (i) Prove that the necessary and sufficient condition for ODE $Mdx + Ndy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(ii) Find the point of inflexion, if any, of the curve $y(a^2 + x^2) = x^3$.

(iii) If $y = (x^2 - 1)^n$, then prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} = n(n+1)y_n$. 4+3+3=10