B.Sc. Semester I (Honours) Examination, 2018-19
MATHEMATICS

Course Id : 12112 **Course Code : SHMTH-102C-2(T)**

Course Title : Algebra

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Use De Moivre's theorem to prove that $\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta + \tan^4\theta}$.

(b) State Division algorithm. Use it to find the remainder when – 326 is divided by 5. 1+1=2

(c) If $S = a + b + c$, prove that $\frac{s}{a-b} + \frac{s}{s-b} + \frac{s}{s-c} > {}^9/_2$.

(d) Find all the equivalence relations on the set $S = \{a, b, c\}$.

(e) For what values of k, the following system of equations has a non-trivial solution?

$$x + 2y + 3z = kx, 2x + y + 3z = ky, 2x + 3y + z = kz$$

(f) When a transformation $T: R^m \to R^n$ is said to be a linear transformation? Is the following transformation linear?

$T: R^2 \to R$ defined by

$T(x, y) = |x - y|$, for all $x, y \in R$.

(g) If an equation $f(x) = 0$ with real coefficients consists of only even powers of x with all positive signs, show with proper reason that the equation cannot have a real root.

(h) Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 2 \end{pmatrix}$. Find the eigenvalues of the matrix $A^5 - I_3$.

2. Answer *any four* questions: 5×4=20

(a) Define rank of a matrix. Find the row reduced echelon form of the matrix

$\begin{pmatrix} 1 & -1 & 2 & 0 & 4 \\ 2 & 2 & 1 & 5 & 2 \\ 1 & 3 & -1 & 0 & 3 \\ 1 & 7 & -4 & 1 & 1 \end{pmatrix}$ and find its rank. 1+3+1=5

(b) (i) If $ax \equiv ay (mod\ n)$ and a is prime to n, then show that $x \equiv y (mod\ n)$.

(ii) Obtain the equation whose roots are the roots of the equation $x^4 - 8x^2 + 8x + 6 = 0$, each diminished by 2. 1+4=5

(c) If a, b, c be positive real numbers such that the sum of any two is greater than the third, then prove that $abc \geq (a + b - c)(b + c - a)(c + a - b)$. Derive when equality occurs. 4+1=5

(d) State the second principle of mathematical induction and using this principle, prove that $(3+\sqrt{5})^n + (3-\sqrt{5})^n$ is divisible by 2^n, for all $n \in N$ (*N* be set of natural numbers). 5

(e) Show that the eigenvalues of a real symmetric matrix are all real. 5

(f) (i) Show that $A = \{(x_1, x_2, x_3, \dots x_n) \in R^n : x_1 + x_2 + \cdots + x_n = 0\}$ is a subspace of R^n. Find the dimension of the subspace A of R^n.

(ii) Is the union of two subspaces of R^n, a subspace of R^n? Justify your answer. 3+2=5

3. Answer *any one* question: 10×1=10

(a) (i) Show that the transformation $T: R^3 \to R^2$ defined by $T(x, y, z) = (z, x + y)$ is linear.

(ii) Let $f: A \to B$ and $g: B \to C$ be two mappings such that $g_{\circ} f: A \to C$ is injective. Is it necessary that g is injective? Justify your answer.

(iii) Solve the cubic equation $x^3 - 9x + 28 = 0$ by Cardon's method. 3+2+5=10

(b) (i) Prove that any partition of a non-empty set S induces an equivalence relation on S.

(ii) Prove or disprove:

"Composition of two linear transformation from $R^m \to R^n$ is linear transformation."

(iii) A linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x_1, x_2, x_3) = (2x_1 + x_2 - x_3, x_2 + 4x_3, x_1 - x_2 + 3x_3)(x_1, x_2, x_3) \in R^3$. Obtain the matrix of *T* relative to the ordered base (0, 1, 1), (1, 0, 1), (1, 1, 0) of $\mathbb{R}^3$. 3+2+5=10

B.Sc. Semester I (Honours) Examination, 2018-19
MATHEMATICS

Course Id : 12114 **Course Code : SHMTH-103GE-1(T)**

Course Title : Calculus, Geometry & Differential Equation

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Find the *n*-th derivative of the function $y = \log(x + a)$.

(b) Solve: $(x + y + 1)dy = dx$

(c) Evaluate: $\int_0^{\frac{\pi}{2}} \cos^4 x \sin^2 x dx$

(d) Find the envelope of the straight line $y = mx + \frac{a}{m}, m$ being the variable parameter $(m \neq 0)$.

(e) Reduce the differential equation $xy' + y = y^2 \log x$ to a linear form.

(f) Find the asymptotes of $x^2 - 4y^2 = 1$.

(g) Find the equation of the sphere through the points (0, 0, 0), (*a*, 0, 0), (0, *b*, 0) and (0, 0, *c*).

(h) Transform the equation $x^2 + 2\sqrt{3}xy - y^2 - 2 = 0$ to axes inclined at 30° to the original axes.

2. Answer *any four* questions: 5×4=20

(a) (i) Find the value of $\lim_{x\to 1}\left(x^{\frac{1}{1-x}}\right)$.

(ii) If $\lim_{x\to 0}\frac{\sin 2x + a\sin x}{x^3}$ is finite, find a and the value of the limit. 2+(2+1)=5

(b) If $I_n = \int \sec^n x dx$, then show that $(n-1)I_n = \tan x \sec^{n-2}x + (n-2)I_{n-2}$ and hence evaluate $\int_0^{\frac{\pi}{4}} \sec^5 x dx$. 3+2=5

(c) (i) Find the order and degree of the differential equation $\sqrt{y + \left(\frac{dy}{dx}\right)^2} = 1 + x$.

(ii) Find the general solution of the differential equation $\frac{dy}{dx} = (x^2 + 1)(y^2 + 1)$. 2+3=5

(d) If $y = \cos(10\cos^{-1}x)$, show that $(1 - x^2)y_{12} = 21xy_{11}$. State Leibnitz's theorem. 4+1=5

(e) Obtain singular solution of the equation $y = px + p - p^2$, where $p = \frac{dy}{dx}$. 5

(f) The radius of a right circular cylinder is 2 and its axis is given by $\frac{x}{1} = \frac{y}{-2} = \frac{z}{2}$; Find the equation of the cylinder. 5

3. Answer *any one* question: 10×1=10

(a) (i) Find the equation of the cone whose vertex is at (1, 0, –1) and which passes through the circle $x^2 + y^2 + z^2 = 4, x + y + z = 1$.

(ii) Find the length of the astroid $x = \cos^3 t, y = \sin^3 t, 0 \le t \le 2\pi$.

(iii) Using L'Hospital Rule, evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$. 4+3+3=10

(b) (i) Solve: $(1 + xy)ydx + (1 - xy)xdy = 0$

(ii) Show that in any conic the sum of the reciprocals of the segments of a focal chord is constant. 5+5=10

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B.Sc. Semester I (General) Examination, 2018-19
MATHEMATICS

Course Id : 12118 **Course Code : SPMTH-101C-1A(T)**

Course Title : Calculus, Geometry & Differential Equation

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Examine the curve $y - \sin x$ regarding its convexity or concavity to the x-axis.

(b) Evaluate: $\lim_{x\to 0} \frac{\sin x - x}{x^3}$

(c) Find the nature of the conic represented by $9x^2 - 6xy + y^2 - 14x - 2y + 12 = 0$

(d) Identify the order and degree of the differential equation $\sqrt{1+(y')^2} = x + 1$

(e) Evaluate: $\int_0^{\frac{\pi}{2}} \cos^4 x dx$

(f) Find the asymptotes of $x^2 - y^2 = 9$

(g) Find an integrating factor of the differential equation $(x^2 + y^2 + 2x)dx + 2ydy = 0$.

(h) Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$.

2. Answer *any four* questions: 5×4=20

(a) Evaluate: $\lim_{x\to 0} (\cos x)^{\cot^2 x}$ 5

(b) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}, |x| < 1$, then show that

(i) $(1 - x^2)y_2 - 3xy_1 - y = 0$

(ii) $(1 - x^2)y_{n+2} - (2n + 3)xy_{n+1} - (n + 1)^2 y_n = 0$ 2+3=5

(c) Solve: $(4x^2y - 6)dx + x^3dy = 0$ 5

(d) Find the surface area generated by revolving the straight line $x = 1 - y, 0 \le y \le 1$ about y axis. 5

(e) Find the asymptotes of $y^2 - x^2 - 2x - 2y - 3 = 0$. 5

(f) What is rotation of axes? What will be the form of the equation $x^2 - y^2 = 4$, if the co-ordinate axes are rotated through an angle $\left(-\frac{\pi}{2}\right)$. 1+4=5

3. Answer *any one* question: 10×1=10

(a) (i) Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.

(ii) Find if there is any point of inflexion on the curve $y - 3 = 6(x - 2)^5$

(iii) Solve: $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$ 4+3+3=10

(b) (i) Find a and b such that $\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$.

(ii) Find the equation of the right circular cylinder whose radius is 1 and x-axis is the axis. 5+5=10

B.Sc. Semester I (Honours) Examination, 2018-19
MATHEMATICS

Course Id : 12111 **Course Code : SHMTH-101C-1(T)**

Course Title : Calculus, Geometry and Differential Equation

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

(a) Evaluate: $\lim_{x\to 1} \frac{\log(1-x)}{\cot(\pi x)}$

(b) Define rectilinear asymptotes of a plane curve.

(c) Find the perimeter of the cardioid $r = a(1-\cos\theta)$

(d) Define Bernoulli's equation. Can it be put in the linear form? Justify your answer.

(e) Solve : $\frac{dy}{dx} + y\frac{d\Phi(x)}{dx} = \Phi(x)\frac{d\Phi(x)}{dx}$

(f) Show that the semi-latus rectum of a conic is a harmonic mean between the segments of any focal choral.

(g) Find the nature of the conicoid: $3x^2 - 2y^2 - 12x - 12y - 6z = 0$.

(h) Prove that $(a-2, {-2}/{e^2})$ is a point of inflexion of the curve $y = (x-a)e^{x-a}$.

2. Answer *any four* questions: 5×4=20

(a) If $J_n = \int \sec^n x\, dx$, then show that $(n-1)J_n = \tan x\, sec_x^{n-2} + (n-2)J_{n-2}$. Hence, find a reduction formula for $\int_0^{\frac{\pi}{4}} \sec^n x\, dx$ and use this formula to evaluate $\int_0^{\frac{\pi}{4}} \sec^5 x\, dx$. 2+1+2=5

(b) Define envelope of a family of curves. Find the condition between a and b so that the envelope of the family of the lines ${x}/{a} + {y}/{b} = 1$ may be the curve $x^p y^q = k^{p+q}$. 2+3=5

(c) (i) Show that if a straight line meets a conicoid in three points, then the straight line lies wholly on the conicoid.

(ii) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. 2+3=5

(d) Find the area common to the circles $r = a\sqrt{2}$ and $r = 2a\cos\theta$. 5

(e) What do you mean by Integrating Factor? Find the integrating factor of the ODE $(xy\sin xy + \cos xy)y dx + (xy\sin xy - \cos xy)x dy = 0$. Hence solve it. 1+1+3=5

(f) If the population of a country doubles in 100 years, in how many years will it treble under the assumption that the rate of increase is proportional to the number of inhabitants? 5

3. Answer *any one* question: 10×1=10

(a) (i) If $y = \frac{\log x}{x}$, prove that $y_n = \frac{d^n y}{dx^n} = \frac{(-1)^n \lfloor n}{x^{n+1}} [log x - 1 - \frac{1}{2} - \frac{1}{3} \dots - \frac{1}{n}]$.

(ii) Find the asymptotes of the curve

$$\left.\begin{aligned} x &= \frac{1}{t^4-1} \\ y &= \frac{t^3}{t^4-1} \end{aligned}\right\}$$

(iii) Find the surface generated by the revolution of an arc of the catenary $y = c \cosh {}^x/_c$ about the axis of x. 3+3+4=10

(b) (i) Find the values of b and g such that the equation $9x^2 + 12xy + by^2 + 2gx + 4y + 1 = 0$ represents a conic having infinitely many centres and determine the nature of the conic.

(ii) A variable plane is parallel to the given plane ${}^x/_a + {}^y/_b + {}^z/_c = 2$ and meets the axes in A, B, C respectively. Show that the circle ABC lies on the cone

$$\left({}^b/_c + {}^c/_b\right)yz + ({}^a/_c + {}^c/_a)zx + \left({}^a/_b + {}^b/_a\right)xy = 0$$

(iii) Show that if y_1 and y_2 are two solutions of the ODE $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x only and $y_2 = y_1 z$ then show that $z = 1 + ae^{-\int \frac{Q}{y_1} dx}$, a being an arbitrary constant. 3+4+3=10
