BANKURA UNIVERSITY

B. Sc. Semester I (Hons) Examination 2017

MATHEMATICS

Subject Code : 12102 Course Code : UG/SC/102/C-02

Course Title : Algebra

Full Marks : 40 **Time : 2 Hours**

The figures in the right hand side margin indicate marks.

1. Answer any five questions : **2x5=10**

a) If the sum of two roots of the equation $x^3 + px^2 + qx + r = 0$ is zero, then show that $pq = r$

b) Express the complex number $-1-i$ in Polar form with Principal argument.

c) z is a variable complex number such that an amplitude of $\frac{z-i}{z+1}$ is $\pi/4$. Show that the point z lies on a circle in the complex plane.

d) Show that for any three real numbers a, b, c

$a^8 + b^8 + c^8 \geq a^2 b^2 c^2 (ab + bc + ca)$.

e) Let λ be an eigen value of an n x n matrix A. Show that λ^2 is an eigen value of A^2.

f) Is the following transformation linear?

$T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$T(x,y) = (x + y, 0), \forall x, y \in \mathbb{R}$

g) Suppose R and S are two equivalence relations on a nonempty set A. Verify whether $R \cup S$ and $R \cap S$ are equivalence relations on A.

h) Use the theory of congruence to find the remainder when the sum $1^3+2^3+3^3+.....+99^3$ is divided by 3.

2. Answer any four questions : **5x4=20**

a) i) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose roots are $\alpha\beta + \beta\gamma,\ \beta\gamma + \gamma\alpha,\ \gamma\alpha + \alpha\beta$. 3

ii) Find the minimum number of complex roots of the equation $x^7 - 3x^3 + x^2 = 0$. 2

b) Determine the values of a and b for which the system 2+2+1

$$x+y+z=1$$
$$x+2y-z=b$$
$$5x+7y+az=b^2$$

has no solution, only one solution, many solutions respectively.

c) If

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix},$$

then find three eigen vectors of A which are linearly independent. 5

d) i) If a is prime to b then prove that a^2+b^2 is prime to a^2b^2. 2

ii) Prove that the product of k consecutive integers is divisible by k. 3

e) A relation β is defined on $\mathbb{Z}$ by "$x \beta y$ if and only if $x^2 - y^2$ is divisible by 5" for $x, y, \in \mathbb{Z}$. Prove that β is an equivalence relation on $\mathbb{Z}$ and find the distinct equivalence classes. 5

f) Let V(F) be a vector space and $S \neq \phi$ be a finite subset of V. Then prove the set W of all linear combinations of the elements of S forms a subspace of V. Also show that W is the smallest subspace of V containing S. 3+2

3. Answer either (a) or (b) : 10 x 1 = 10

a) i) Show that the sets $A=\{x \in \mathbb{R} : 0 \leq x \leq 1\}$ and $B = \{x \in \mathbb{R} : a \leq x \leq b\}$ have same cardinal number. 3

ii) If $|z| = 1$ and amp $z = \theta$, $(0 < \theta < \pi)$ then find the modulus and principal amplitude of $\frac{2}{1+z}$. 4

iii) Prove that for $n > 3$, the integers n, $n + 2$, $n + 4$ cannot be all primes. 3

b) i) State and prove the Cauchy - Schwarz inequality. 6

ii) Verify Caley - Hamilton theorem for the matrix 4

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}.$$ Hence find A^{-1}.

BANKURA UNIVERSITY
B. Sc. Semester I (Hons) Examination 2017
MATHEMATICS

Subject Code : 12101 Course Code : UG/SC/101/C-01

Course Title : Calculus, Geometry & Differential Equation

Full Marks : 40 **Time : 2 Hours**

The figures in the right hand side margin indicate marks.

1. Answer any five questions : **2x5=10**

a) Evaluate $\lim_{x \to \frac{\pi}{2}} \left(x \tan x - \frac{\pi}{2} \sec x \right)$.

b) Find the envelope of the curves $x^2 \cos\theta + y^2 \sin\theta = a^2$, θ is a parameter.

c) Find the length of one arc of the cycloid

$x = a(t - \sin t),\ y = a(1 - \cos t),\ a \geq 0.$

d) Show that the conic $ax^2 + 2hxy + by^2 + 2gx\cos^2\alpha + 2fy\sin^2\alpha + c = 0$, where α is a parameter, always passes through two fixed points, provided $g^2 f^2 > c(af^2 + 2hfg + bg^2)$.

e) Show that $I(x, y) = e^{\int p(x)dx}$ is an integrating factor for $\frac{dy}{dx} + p(x)y = 0$, where $p(x)$ denotes an integrable function.

f) If $y = \sin 3x \, . \, \cos 2x$, find y_n, where the suffix n denotes the order of differentiation with respect to x.

g) Assume that the population $x(t)$ of bacteria present in a nutrient solution at any time t is increasing at a rate proportional to the population at that time, find x as a function of t, if the initial population being given as x_0 at $t = 0$.

h) Find the equation of the circle on the sphere $x^2 + y^2 + z^2 = 49$ whose centre is at the point $(2, -1, 3)$.

2. Answer any four questions : **5x4=20**

a) State Leibnitz's Theorem on successive derivatives. 1+4

If $x = \tan(\log y)$ prove that

$(1 + x^2) y_{n+1} + (2nx - 1) y_n + n(n-1) y_{n-1} = 0$

b) Find all the asymptotes of the curve 5

$x^3+2x^2y-xy^2-2y^3+xy-y^2-1=0.$

c) If $I_n=\int_0^{\frac{\pi}{2}} x^n \sin x\, dx$, n being a positive integer >1 2+3

show that $I_n+n(n-1)\, I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$.

Hence find the value of $\int_0^{\frac{\pi}{2}} x^5 \sin x\, dx$.

d) Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic $\frac{l}{r}=1+e\cos\theta$, if $(l\cos\alpha-ep)^2+l^2\sin^2\alpha=p^2$. 5

e) Find the equations of the lines in which the plane $2x-6y-5z=0$ cuts the cone $yz+zx+xy=0$. 5

f) i) Solve the logistic model $\dot{x}=kx(1-x)$, $k>0$ subject to the condition $x=x_0$ at $t=t_0$ and show that $x\to 1$ as $t\to\infty$.

ii) Solve : $x\cos\left(\frac{y}{x}\right)(ydx+xdy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx)$. (2+1)+2

3. Answer <u>any one</u> question : 10 x 1 = 10

a) i) Find the nature of the quadric surface

$2x^2+5y^2+3z^2-4x+20y-6z=5$ 2

ii) Find the equation of the right circular cylinder of radius 3 and whose axis is $\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z-3}{6}$. 3

iii) The arc of the cardiod $r=a(1+\cos\theta)$ specified

by $-\pi/2\le\theta\le\pi/2$, is rotated about the line $\theta=0$. Find the area of the generated surface of revolution. 5

b) i) If $I_n=\int_0^{\frac{\pi}{4}}\tan^n x\, dx$, then prove that $I_n=\frac{1}{n-1}-I_{n-2}$. 4

ii) Solve :

(I) $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$, given that $y(0)=\pi/4$. 3

(II) $\frac{dy}{dx}+\frac{y}{x}\log_e y=\frac{y}{x^2}(\log_e y)^2$. 3

BANKURA UNIVERSITY
B. Sc. Semester I (Programme) Examination 2017
MATHEMATICS

Subject Code : 12104 Course Code : UGP/SC/101/C-1A

Course Title : Calculus, Geometry & Differential Equation

Full Marks : 40 **Time : 2 Hours**

The figures in the right hand side margin indicate marks.

1. Answer any five questions : 2x5=10

a) Using L' Hospital's rule show that $\lim_{x\to\frac{\pi}{2}} \frac{\tan x - x}{x - \sin x} = 2$ 2

b) Find the oblique asymptote of the curve $y = xe^{\frac{1}{x}}$ 2

c) Determine the length of the parametric curve given by the following parametric equations $x = 3\sin t$ and $y = 3\cos t$; $0 \le t \le 2\pi$. 2

d) Find the order and degree of the following differential equation.

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} = 2\frac{d^2y}{dx^2}$$ 2

e) Solve $(1-x^2)\frac{dy}{dx} - 2xy = x - x^3$. 2

f) Find the centre and the radius of the sphere 2

$3x^2+3y^2+3z^2+2x-4y-2z-1=0$

g) Find the nature of the quadric surface given by the equation 2

$2x^2+5y^2+3z^2-4x+20y-6z=5$

h) Evaluate $\int_0^{\frac{\pi}{4}} \tan^6 x\, dx$. 2

2. Answer any four questions : 5x4=20

a) Sketch the curve parameterized by 5

$r(t) = <1-t^2, 2t^2-t^4>, t \in [0, 1]$

b) Find the values of a and b such that 5

$$\lim_{x\to 0} \frac{x(1-a\cos x)+b\sin x}{x^3} = \frac{1}{3}$$

(Assume that L' Hospital's rule is applicable)

c) Show that $\int_0^{\frac{\pi}{2}} \sin^n x\, dx = \frac{(n-1)(n-3)(n-5).....1}{n(n-2)(n-4).....2} \cdot \frac{\pi}{2}$ 5

if n be any even positive integer and $n > 1$.

d) If $y=\left(x+\sqrt{1+x^2}\right)^m$, then find the value of $y_n(0)$. 5

e) Find the equation of the sphere described on the join of $p(2,-3,4)$ and $\theta(-5,6,-7)$ as diameter. 5

f) Solve : $\frac{dy}{dx}+\frac{4x}{x^2+1}y=\frac{1}{(x^2+1)^3}$ 5

3. Answer <u>any one</u> question : 10 x 1 = 10

a) i) Find the envelope of the family of co-axial ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where the parameter a and b are connected by $a^n+b^n=c^n$. 6

ii) Find the asymptotes of $x^3+x^2y-xy^2-y^3+x^2-y^2=2$. 4

b) I If $I_n=\int_0^{\frac{\pi}{2}} x\sin^n x\, dx, n>1$, show that $I_n=\frac{n-1}{n}\quad I_{n-2}+\frac{1}{n^2}$.

Hence evaluate $\int_0^{\frac{\pi}{2}} x\sin^5 x\, dx$. 3+2=5

ii) Solve : $(x^2y^3+2xy)\,dy=dx$. 5

BANKURA UNIVERSITY

B. Sc. Semester I (Hons) Examination 2017

MATHEMATICS

Subject Code : 12103 Course Code : UG/SC/103/GE-01

Course Title : Calculus, Geometry & Differential Equation

Full Marks : 40 **Time : 2 Hours**

The figures in the right hand side margin indicate marks.

1. Answer <u>any five</u> questions : **2x5=10**

a) Define general solution and particular solution of the differential equation. 1+1

b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^5 x \, dx$. 2

c) State L' Hospital Rule. 2

d) Check the curve $y=\sqrt{1+x^2}\sin\left(\frac{1}{x}\right)$ has vertical asymptotes or not in the neighbourhood of $x=0$. 2

e) Solve : $\frac{dy}{dx}=e^{(x+y)}+x^2e^{(x^3+y)}$. 2

f) Find the envelope of $y=mx+\sqrt{a^2m^2+b^2}$. where in v the variable parameter. 2

g) Find the nature of the quadric surface given by the equation $2x^2+5y^2+3z^2-4x+20y-6z=5$. 2

h) Find the equation of the sphere which passes through the origin and touches the sphere $x^2+y^2+z^2=56$ at the point (2, –4, 6). 2

2. Answer <u>any four</u> questions : **5x4=20**

a) Find the equation of the cylinder whose generating line is parallel to the z-axis and the guiding curve

$x^2+y^2=z,\ x+y+z=1.$ 5

b) Sketch the Cartesian equation of the curve $r(\theta)=<2\sin\theta\cos\theta, 2\sin^2\theta>,\ \theta\in[0,\pi]$. 5

c) What do you mean by Integrating Factor ? Find the integrating Factor of the differential equation $(x^2y-2xy^2)\,dx+(3x^2y-x^3)\,dy = 0$, hence solve it. 1+1+3

d) Show that $\int_0^{\frac{\pi}{2}} \cos^n x\, dx=\frac{(n-1)(n-3)(n-5).....3.1}{n(n-2)(n-4).....4.2}\ \frac{\pi}{2}$

if n be any even positive integer, $n > 1$. 5

e) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then prove that

$(x^2 - 1) y_{n+2} + (2n+1) x y_{n+1} + (n^2 - m^2) y_n = 0$ 5

f) Solve : $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$. 5

3. Answer <u>any one</u> question : 10 x 1 = 10

a) i) Define exact differential equation.

Solve $(3x^2y^4 + 2xy)\, dx\, (2x^3y^3 - x^2)\, dy = 0$ 1+4=5

ii) If $J_n = \int_0^{\frac{\pi}{4}} \tan^n x\, dx$ then prove that $J_n = \frac{1}{n-1} - J_{n-2}$, where n be positive integer, $n > 1$. Hence evaluate $\int_0^{\frac{\pi}{4}} \tan^6 x\, dx$. 3+2=5

b) i) Find the asymptotes of the curve $x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$. 5

ii) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and which touches the plane $2x + 2y - z = 15$. 5