SH-I/Forestry/BS/1104/19

B.Sc. 1st Semester (Honours) Examination, 2019-20

FORESTRY

Course ID : BS1104 **Course Code : SH/BS/1104**

Course Title: Basic Mathematics

Time: 3 Hours **Full Marks: 70**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meaning.

Answer *any twenty* questions. 1×20=20

1. (a) If x and y are real numbers and $x - iy = 0, i = \sqrt{-1}$, find the values of x and y.

(b) If every term of an AP is increased by 3, Prove that the numbers thus obtained also form an AP.

(c) A polygon has 44 diagonals, find the number of sides of the Polygon.

(d) Find the middle term of the expansion of $\left(x + \frac{1}{x}\right)^8$.

(e) Prove that $sin\,_x^{-1} + cos_x^{-1} = \frac{\pi}{2}$.

(f) Find the value of $\sin\left(-\frac{11}{4}\pi\right)$.

(g) The third term of a 6P is 4. Find the product of its first five terms.

(h) Find the square root of $-8i,\ i = \sqrt{-1}$.

(i) Evaluate $\int \sin 3x \cos x \, dx$.

(j) What is the order of the matrix A^T where $A = [1\ 5\ 9\ 4\ 2]$.

(k) Without expanding find the value of the determinant $A = \begin{vmatrix} 2 & 3 & 5 \\ 4 & 6 & 10 \\ -1 & 2 & 3 \end{vmatrix}$.

(l) If $y = xe^x$ show that $x\frac{dy}{dx} = (1 + x)y$.

(m) Evaluate $\lim\limits_{x \to 0} \frac{\sqrt{1+x^2}-1}{x^2}$.

(n) In a triangle ABC given $\angle A = 60°, \angle B = 45°$ and $a = 2\sqrt{3}$ unit find b where a and b have the usual meaning.

(o) Calculate the total no. of permutations with the letters of BANANA taking all together.

(p) There lie thirteen arithmetic means between the numbers 10 and 52. Obtain the common difference of the A.P.

(q) If in a triangle ABC, $\frac{a-b+c}{a} = \frac{b}{b+c-a}$, prove that $\underline{|c} = 60°$.

(r) If $y = x + \frac{1}{x}$ find the points where $\frac{dy}{dx} = 0$.

(s) If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 1 \\ -1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & 2 \\ -1 & 2 & 0 \\ -3 & -1 & 1 \end{bmatrix}$ then find X such that A + X = B

(t) If $f(x) = \begin{matrix} 5x - 4 & when & 0 < x \leq 1 \\ 4x^3 - 3x & when & 1 < x < 2 \end{matrix}$'

verify whether $\lim_{x\to 1} f(x)$ exists.

(u) Find the value of $\int x^2 \log x \, dx$.

(v) Test whether the function $f(x) = x^3 - 6x^2 + 24x + 4$ has any maximum or minimum value with justification.

(w) $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ find $A - 3I_2$ with usual meaning of I_2.

(x) Using binomial theorem find the value of 9^5.

(y) Prove geometrically $\tan 45° = 1$

(z) Prove that $\cos 306° + \cos 234° + \cos 162° + \cos 18° = 0$.

(aa) If $\cos^{-1}\frac{1}{\sqrt{5}} = \theta$, find $cosec^{-1}\sqrt{5}$.

(bb) If $\sin 3\propto = \cos 3\beta$ where α and β are both acute angles, find $\sin(\alpha + \beta)$.

(cc) For what value of *k*, the system of equations $\begin{matrix} x + 2y = 3 \\ 3x + ky = 7 \end{matrix}$ has no solution?

(dd) Write the expression for the sum of 1st *n* terms of the series $1^2 + 2^2 + 3^2 + \cdots$.

2. Answer *any ten* questions: 2×10=20

(a) If *a, b, c* are in AP, show that $(a + 2b - c)(2b + c - a)(c + a - b) = 4abc$.

(b) Find a complex number *z* so that $z(3 + 4i) = 2 + 3i$.

(c) Prove that $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

(d) Examine whether there exists any term involving x^6 in the expansion of $\left(x + \frac{1}{x^2}\right)^n$.

(e) If in an examination an examinee has to obtain a minimum marks in each of 5 subjects, calculate in how many ways he will become unsuccessful.

(f) Find the value of x if $\sin^{-1}\cos\sin^{-1} x = \frac{\pi}{3}$.

(g) If in a triangle ABC $\sin A : \sin C = \sin(A - B) : \sin(B - C)$ then prove that a^2, b^2, c^2 are in AP.

(h) Find the value of $\lim_{x\to 0} \frac{2\sin x - \sin 2x}{x^3}$.

(i) If $X = \begin{bmatrix} 2 & 3 \\ -5 & 7 \\ -2 & 3 \end{bmatrix}, Y = \begin{bmatrix} 9 & -5 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$, find 2X + 3Y.

(j) If $f(x) = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$, obtain $f'(x)$.

(k) If $x \cdot {}^{n}c_r = {}^{n}p_r$, find *x* where notations are usual.

(l) Taking suitable matrices, show that matrix multiplication is not commutative.

(m) Find explicitly the fourth term of the series $1 + \left(\frac{1}{2^p} + \frac{1}{3^p}\right) + \left(\frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p}\right) + \cdots$.

(n) Express $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ -5 & 6 & 7 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

(o) With usual notations, prove that ${}^{n}C_r + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2} = {}^{n+2}C_r$.

3. Answer *any five* questions: 6×5=30

(a) Find the sum to first *n* terms of the series $4 + 44 + 444 + \cdots$. 6

(b) With usual notations prove that ${}^{2n}P_n = 2^n\{1{\cdot}3{\cdot}5 \ldots (2n-1)\}$. 6

(c) If $y = \sqrt{x}$, find $\frac{dy}{dx}$ by definition. 6

(d) Obtain the maximum and minimum values of $f(x) = x^3 - 9x^2 + 15x - 3$ 3+3=6

(e) If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$

Find the value A $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$. 6

(f) Obtain the Adjoint and inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ if it exists. 4+2=6

(g) (i) Represent the following system A linear equations $x + y += 4,\ y - z = 1,\ 2x + y + 4z = 7$ in matrix notations. Test whether system is consistent.

(ii) When a system of linear equations is said to have infinitely many solutions? (3+1)+2=6

(h) The circum radius of a triangle is 10 cm and the angles are in the ratio 2 : 3 : 7, find the sides of a triangle. 6

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