B.Sc. 1st Semester(Honours) Examination-2021

FORESTRY

Course ID:13508 **Course Code: SH/BS/FST/BS 1104**

Course Title: Basic Mathematics

Time: 3 Hours **Full Marks: 70**

I. Answer ***any twenty*** questions: $1\times 20 = 20$

1. Find the sum of numbers of the series $3 + 3^2 + 3^3 + \cdots$ without any formula.
2. Let ω be a complex cube root of 1. Without using any formula, prove that $(1-\omega^2)(1-\omega^4)(1-\omega^8)(1-\omega^{10}) = 9$.
3. Obtain the total number of permutations with the letter of the word 'TRIANGLE'.
4. The nth term of a G.P is 2^{2n-1}.Find the sum of its first four terms.
5. Taking a suitable example prove that if any two columns of determinant of order 3 are identical,the value of its is zero.
6. Find the square root of $5 + 2\sqrt{6}$.
7. In a triangle $\Delta ABC, A = 3B \; and \; a = 2b$ with usual meaning, obtain the angles of the triangles.
8. Find the co-efficient of x^min the expansion of $(1+x)^{m+n}$.
9. Integrate the function $sin^3 x$ w.r.to x.
10. Show that $\cos 15° - \sin 15° = \frac{1}{\sqrt{2}}$.
11. If $z = x + iy \; and \; az + ib\bar{z} = 0$. Find the value $x + y$ where $a + b \neq 0 \; and \; \overline{z}$ is
12. the conjugate of z.
13. If the numbers a, b, c are in A.P, show that $b + c - a, c + a - b, a + b - c$ are also in A.P.

14. If $f(x+3) = 2x^2 - 3x + 1$,obtain $f(x+1)$.

15. Find the domain of the function $f(x) = \frac{|x|}{x}$.

16. Evaluate $\lim_{x\to 0} \frac{1-cos^3 x}{tan^2 x}$.

17. Let $\begin{pmatrix} x & 5 \\ 7 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ y & 1 \end{pmatrix}$, *find the value of* x & y.

Please Turn Over

18. Prove that $\cos(\frac{\pi}{4} - x) + \cos(\frac{\pi}{4} + x) = \sqrt{2}\cos x$.

19. If $\sin x = {}^{3}/_{5}, \cos y = {}^{9}/_{41}$ *and* x, y *are acute angles.* Find the value of Sin $(x - y)$.

20. The roots of the equation $x(x-3) = 4$ *are* α & β. Find the value of $\alpha^2 + \beta^2$.

21. Eliminate θ from the equations $x = a\sec\theta$, and $y = b\tan\theta$.

22. If $\sin^{-1}\cos\sin^{-1} x = \frac{\pi}{3}$, find the value of x.

23. Express $\sin\theta$ in terms of$\tan\frac{\theta}{2}$.

24. If ${}^{n}_{6}c = {}^{n-2}_{4}c$, *find* ${}^{n}_{3}c$.

25. If $x = a(t + \sin t), y = a(1 - \cos t)$, *find* $\frac{dy}{dx}$.

26. Establish geometrically the formula $sin^2\theta + cos^2\theta = 1$

27. State when a function$f(x)$ has neither a maximum nor a minimum.

28. Write the 5th term of the series$x + \frac{1}{2}.\frac{x^3}{3} + \frac{1.3}{2.4}.\frac{x^5}{5} + \frac{1.3.5}{2.4.6}.\frac{x^7}{7}$+…….

29. For what value of '$\boldsymbol{a}$', 2 is a root of the equation $x^2 - 3ax + 2 = 0$.

30. Let $A = \begin{pmatrix} 2 & 5 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ form a matrix B so that AB is possible. Also compute AB.

31. Evaluate $\lim_{x \to 3} \frac{x^3 - 9}{x - 3}$.

II. Answer ***any ten*** questions: $2 \times 10 = 20$

1. Express the complex numbers i and $-i$ in the polar form of $\cos\theta + i\sin\theta$.

2. Expand $(x - \frac{1}{2x})^5$.

3. If $x = \cos 55°$, $y = \cos 65°$ *and* $z = \cos 175°$, show that $x + y + z = 0$.

4. How many numbers containing 3 digits can be formed from $0, 1, 2, 3, 4$ & 6.

5. In an A.P the 5^{th} and 20^{th} terms are 6 and -69 respectively. Obtain the series.

6. From definition , find the derivative of $\tan x$.

Please Turn Over

7. When does the function $(\sin 3x - 3\sin x)$ attain its maximum value or minimum value in $(0, 2\pi)$?
8. In a G.P 3 + 6 + 12 + …….. , the sum of first n terms is 381. Find the value of n.
9. Find the middle term of the expression $(x - \frac{1}{x})^8$.
10. Establish the formula $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$.
11. In a plane, 4 points are collinear out of 10 points. Find the no. of
    (i) straight lines and
    (ii) tringles can be drawn joining the points.

12. Evaluate $\int \sqrt{1 - x^2}\, dx$; $|x| < 1$.
13. In triangle ΔABC, prove that $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$, where a,b,c has the usual meaning and $C = 90°$.

14. Verify whether the system of linear equations $3x + 4y = 7, and\ 6x + 8y = 13$ is consistent.

15. Prove that $\sin^{-1}\frac{3}{5} + \text{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$.

III. Answer **any six** questions $6\times 5 = 30$

1. Find the matrix X from the equation $AX = B$ where $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$ and B=$\begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$.
2. Prove that the determinant $\begin{vmatrix} 2ab & a^2 & b^2 \\ a^2 & b^2 & 2ab \\ b^2 & 2ab & a^2 \end{vmatrix} = -(a^3 + b^3)^2$.
3. A group of 5 members is to form out of 6 men and 4 women. In how many ways this group can be formed if each group consists of at least one woman?
4. If $x = -1 + i\sqrt{2}$, find the value of $x^4 + 4x^3 + 6x^2 + 4x + 11$.
5. If the numbers a, b, c are in A.P and x, y, z are in G.P.Show that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$.
6. Show that the rectangle inscribed in a circle has maximum area when it is a square.

Please Turn Over

7. Evaluate (i) $\lim_{x\to 0} \frac{x^2 \sin\frac{1}{x}}{\sin x}$.

   (ii) $\lim_{x\to 1} \frac{x^2-3x+2}{x^2-4x+3}$.

8. For what value/values of **a** the following system of linear equations are consistent ?

$$x - y + z = 1$$
$$x + 2y + 4z = a$$
$$x + 4y + 6z = a^2$$